C3-II laskarit vko 49

1.

$$y' = Ay, \quad A = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}$$

$$\det(A - I\lambda) = \lambda^2 + 4 = 0 \qquad \lambda = \pm 2i$$

$$\lambda = 2i: \quad \begin{pmatrix} -2i & -2 \\ 2 & -2i \end{pmatrix} x = 0 \quad \Rightarrow \quad x = \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

$$\lambda = -2i: \quad \begin{pmatrix} 2i & -2 \\ 2 & 2i \end{pmatrix} x = 0 \quad \Rightarrow \quad x = \begin{pmatrix} 1 \\ i \end{pmatrix}$$

Kreyszig menetelmä:

$$y(t) = c_1 \begin{pmatrix} 1 \\ -i \end{pmatrix} e^{2it} + c_2 \begin{pmatrix} 1 \\ i \end{pmatrix} e^{-2it}$$

$$= c_1 \begin{pmatrix} 1 \\ -i \end{pmatrix} (\cos 2t + i \sin 2t) + c_2 \begin{pmatrix} 1 \\ i \end{pmatrix} (\cos 2t - i \sin 2t)$$

$$= c_1 \begin{pmatrix} \cos 2t + i \sin 2t \\ -i \cos 2t + \sin 2t \end{pmatrix} + c_2 \begin{pmatrix} \cos 2t - i \sin 2t \\ i \cos 2t + \sin 2t \end{pmatrix}$$

$$= (c_1 + c_2) \begin{pmatrix} \cos 2t \\ \sin 2t \end{pmatrix} + i(c_1 - c_2) \begin{pmatrix} \sin 2t \\ -\cos 2t \end{pmatrix}$$

merk $d_1 = c_1 + c_2$, $d_2 = i(c_1 - c_2)$

$$= d_1 \begin{pmatrix} \cos 2t \\ \sin 2t \end{pmatrix} + d_2 \begin{pmatrix} \sin 2t \\ -\cos 2t \end{pmatrix}$$

Matriisieksponentti:

$$X = \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix} \quad \Rightarrow \quad X^{-1} = \frac{1}{2} \begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}$$

$$e^{At} = X e^{Dt} X^{-1}$$

$$e^{At} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix}\begin{pmatrix} e^{2it} & 0 \\ 0 & e^{-2it} \end{pmatrix}\begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix}\begin{pmatrix} e^{2it} & ie^{2it} \\ e^{-2it} & -ie^{-2it} \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} e^{2it} + e^{-2it} & ie^{2it} - ie^{-2it} \\ -ie^{2it} + ie^{-2it} & e^{2it} + e^{-2it} \end{pmatrix}$$

$$= \begin{pmatrix} \cos 2t & -\sin 2t \\ \sin 2t & \cos 2t \end{pmatrix}$$

yleinen ratkaisu

$$y(t) = e^{At}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = c_1\begin{pmatrix} \cos 2t \\ \sin 2t \end{pmatrix} + c_2\begin{pmatrix} -\sin 2t \\ \cos 2t \end{pmatrix}$$

2.

Origo on systeemin $x' = Ax$ nielu, jos A:n om-arvojen reaaliosat ovat negatiiviset.

a) $A = \begin{pmatrix} a & -k \\ k & 2 \end{pmatrix}$

$\det(A - I\lambda) = (a-\lambda)(2-\lambda) + k^2 = 0$

$\lambda^2 - (2+a)\lambda + 2a + k^2 = 0$

$$\lambda = \frac{2 + a \pm \sqrt{(2+a)^2 - 4(2a+k^2)}}{2 \cdot 1}$$

$\operatorname{Re}\lambda < 0 \Rightarrow 2 + a < 0$ ja $(2+a)^2 > (2+a)^2 - 4(2a+k^2)$

$\underline{a < -2}$ $\quad -4(2a+k^2) < 0$

$k^2 > -2a$

$\underline{|k| > \sqrt{-2a}}$

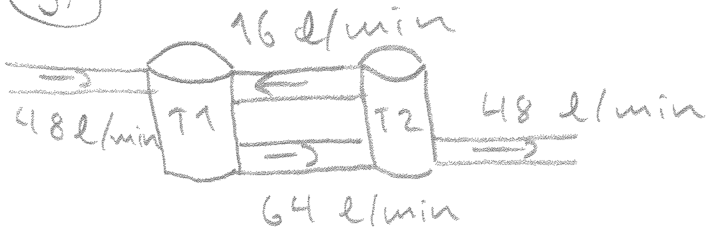
b) $\det(A - I\lambda) = \begin{vmatrix} -\lambda & -1 & 0 \\ 1 & -\lambda & 0 \\ -1 & 0 & k-\lambda \end{vmatrix}$

$= \lambda^2(k-\lambda) + k - \lambda = 0$

$(k-\lambda)(\lambda^2+1) = 0$

$\Rightarrow \lambda_1 = k,\ \lambda_2 = i,\ \lambda_3 = -i \Rightarrow$ origo ei koskaan nielu

3.



alussa

T1: 400 l, 100 kg, T2: 400 l, 40 kg

$y_1(t)$ suolan määrä (kg) tankissa T1

$y_2(t)$ 〃 T2

suolan muutokset ajassa dt: (−ulos +sisään)

$$dy_1 = -\frac{64}{400} y_1 dt + \frac{16}{400} y_2 dt$$

$$dy_2 = \frac{64}{400} y_1 dt - \frac{64}{400} y_2 dt$$

$$\Rightarrow \quad y' = \frac{1}{400}\begin{pmatrix} -64 & 16 \\ 64 & -64 \end{pmatrix} y$$

$$y' = \frac{1}{25}\underbrace{\begin{pmatrix} -4 & 1 \\ 4 & -4 \end{pmatrix}}_{=:\tilde{A}} y$$

$$Ax = \lambda x$$
$$\frac{1}{25}\tilde{A} = \lambda x$$
$$\tilde{A} = \underbrace{25\lambda}_{=\tilde{\lambda}} x$$

$$\det(\tilde{A} - I\tilde{\lambda}) = (-4-\tilde{\lambda})^2 - 4 = 0$$
$$-4 - \tilde{\lambda} = \pm 2$$
$$\tilde{\lambda} = -4 \pm 2 \Rightarrow \tilde{\lambda}_1 = -2,\ \tilde{\lambda}_2 = -6$$

$$\tilde{\lambda} = -2: \begin{pmatrix} -2 & 1 \\ 4 & -2 \end{pmatrix} x = 0 \quad \Rightarrow \quad x = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\tilde{\lambda} = -6: \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix} x = 0 \quad \Rightarrow \quad x = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$X = \begin{pmatrix} 1 & 1 \\ 2 & -2 \end{pmatrix} \Rightarrow X^{-1} = \frac{1}{4}\begin{pmatrix} 2 & 1 \\ 2 & -1 \end{pmatrix}$$

$$y(t) = e^{At} = X e^{Dt} X^{-1} y(0)$$
$$= \frac{1}{4}\begin{pmatrix} 1 & 1 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} e^{-\frac{2}{25}t} & 0 \\ 0 & e^{-\frac{6}{25}t} \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 100 \\ 40 \end{pmatrix}$$
$$= \begin{pmatrix} 60 e^{-\frac{2}{25}t} + 40 e^{-\frac{6}{25}t} \\ 120 e^{-\frac{2}{25}t} - 80 e^{-\frac{6}{25}t} \end{pmatrix}$$

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In[43]:= A = (1 / 25) {{-4, 1}, {4, -4}}
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Out[43]= $\left\{\left\{-\frac{4}{25}, \frac{1}{25}\right\}, \left\{\frac{4}{25}, -\frac{4}{25}\right\}\right\}$

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In[53]:= Plot[Evaluate[MatrixExp[A t].{100, 40}], {t, 0, 60}]
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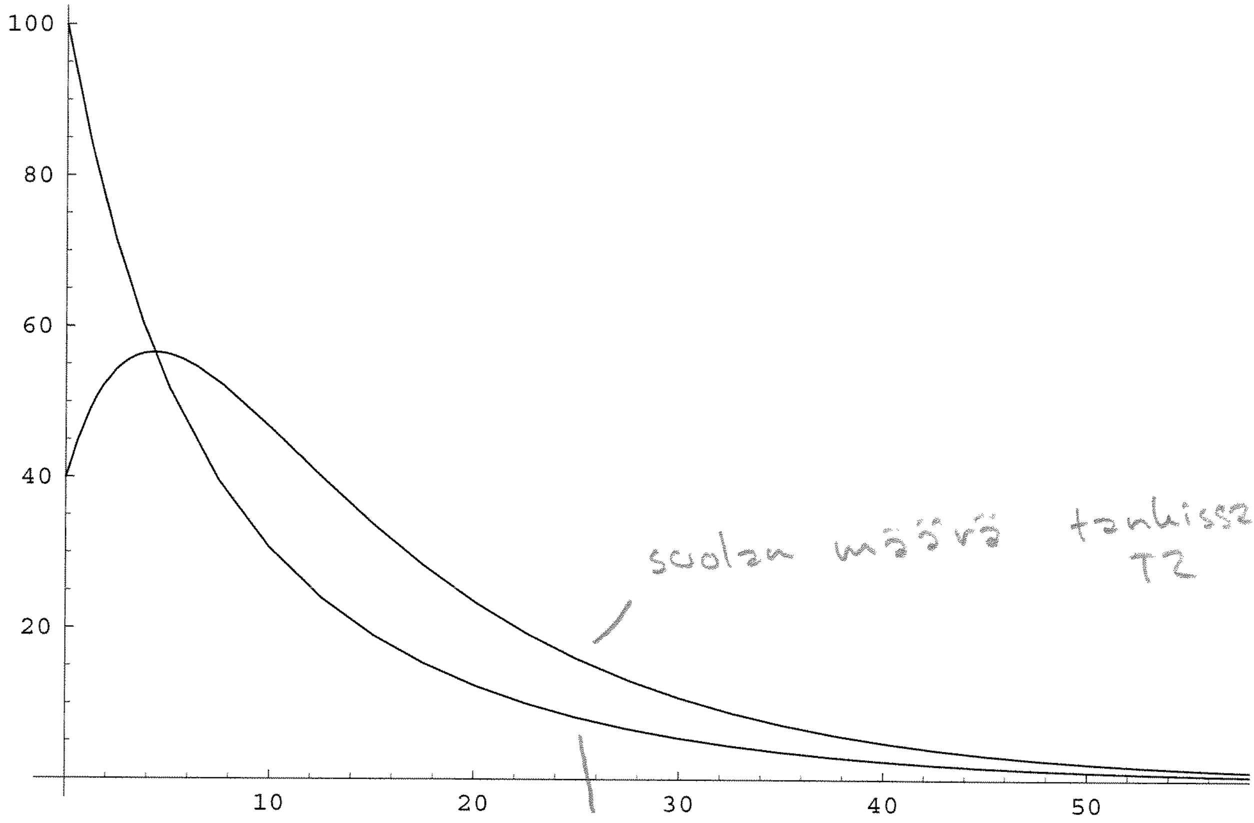


Out[53]= - Graphics -

L1) $y' = \begin{pmatrix} 0 & -2 \\ 8 & 0 \end{pmatrix} y$

$\det(A - I\lambda) = \lambda^2 + 16 = 0 \quad \Rightarrow \quad \lambda_1 = 4i,\ \lambda_2 = -4i$

$\lambda = 4i: \quad \begin{pmatrix} -4i & -2 \\ 8 & -4i \end{pmatrix} x = 0 \quad \Rightarrow \quad x = \begin{pmatrix} 1 \\ -2i \end{pmatrix}$

$\lambda = -4i: \quad \begin{pmatrix} 4i & -2 \\ 8 & -4i \end{pmatrix} x = 0 \quad \Rightarrow \quad x = \begin{pmatrix} 1 \\ 2i \end{pmatrix}$

$X = \begin{pmatrix} 1 & 1 \\ -2i & 2i \end{pmatrix} \quad \Rightarrow \quad X^{-1} = \frac{1}{4} \begin{pmatrix} 2 & i \\ 2 & -i \end{pmatrix}$

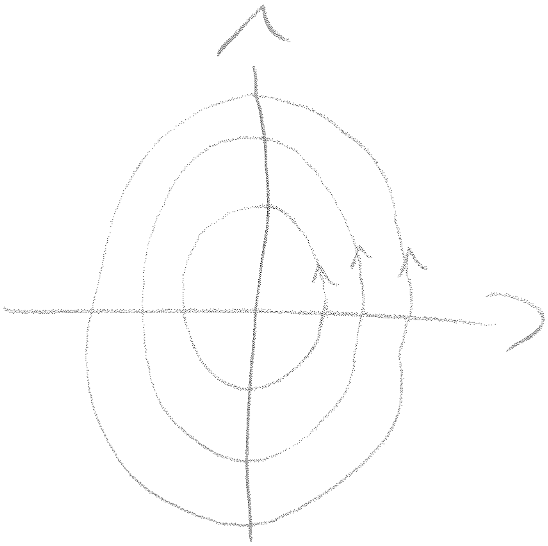
$$e^{At} = X e^{Dt} X^{-1} = \frac{1}{4} \begin{pmatrix} 1 & 1 \\ -2i & 2i \end{pmatrix} \begin{pmatrix} e^{4it} & 0 \\ 0 & e^{-4it} \end{pmatrix} \begin{pmatrix} 2 & i \\ 2 & -i \end{pmatrix}$$

$$= \frac{1}{4} \begin{pmatrix} 1 & 1 \\ -2i & 2i \end{pmatrix} \begin{pmatrix} 2e^{4it} & ie^{4it} \\ 2e^{-4it} & -ie^{-4it} \end{pmatrix}$$

$$= \frac{1}{4} \begin{pmatrix} 2e^{4it} + 2e^{-4it} & ie^{4it} - ie^{-4it} \\ -4ie^{4it} + 4ie^{-4it} & 2e^{4it} + 2e^{-4it} \end{pmatrix}$$

$$= \begin{pmatrix} \cos 4t & -\frac{1}{2} \sin 4t \\ 2 \sin 4t & \cos 4t \end{pmatrix}$$

$$y(t) = e^{At} \cdot \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = c_1 \begin{pmatrix} \cos 4t \\ 2 \sin 4t \end{pmatrix} + c_2 \begin{pmatrix} -\frac{1}{2} \sin 4t \\ \cos 4t \end{pmatrix}$$



stabiili keskus

L2) a)

$$p(\lambda) = \det(A - I\lambda) = a_n\lambda^n + a_{n-1}\lambda^{n-1} + \dots + a_1\lambda + a_0$$
$$p(0) = \det(A) = a_0$$

b) Karakteristisen polynomin nollakohdat ovat ominaisarvoja, ja polynomi voidaan jakaa ensimmäisen asteen tekijöihin, joten

$$p(\lambda) = c(\lambda - \lambda_1)(\lambda - \lambda_2)\cdot \ldots \cdot(\lambda - \lambda_n)$$

toisaalta

$$p(\lambda) = \det(A - I\lambda) = \begin{vmatrix} a_{11}-\lambda & \cdots & a_{1n} \\ \vdots & a_{22}-\lambda & \vdots \\ & \ddots & \\ a_{n1} & \cdots & a_{nn}-\lambda \end{vmatrix}$$

$$= (a_{11}-\lambda)(a_{22}-\lambda)\cdot \ldots \cdot(a_{nn}-\lambda) + \dots$$

Determinantti koostuu summasta n-tekijän tuloista, jossa tulon tekijöistä mitkään kaksi eivät ole matriisin samalta riviltä tai sarakkeelta.

Tästä seuraa että $(a_{11}-\lambda)(a_{22}-\lambda)\cdot \ldots \cdot(a_{nn}-\lambda)$ on summan ainoa termi jossa on tekijänä yli n-2 kpl λ:ja. Joten termi määrää sekä λ^n että λ^{n-1} kertoimen.

$$(a_{11}-\lambda)(a_{22}-\lambda)\cdot \ldots \cdot(a_{nn}-\lambda)$$
$$= (-\lambda)^n + (a_{11}+a_{22}+\dots+a_{nn})(-\lambda)^{n-1} + \dots$$
$$= (-1)^n\lambda^n + (a_{11}+a_{22}+\dots+a_{nn})(-1)^{n-1}\lambda^{n-1} + \dots$$

Verrataan $p(\lambda)$:n tekijämuotoon

$p(\lambda) = c(\lambda - \lambda_1)(\lambda - \lambda_2) \cdot \ldots \cdot (\lambda - \lambda_n)$

$= c\lambda^n + c(-\lambda_1 - \lambda_2 - \ldots - \lambda_n)\lambda^{n-1} + \ldots$

Vertaamalla λ^n kerrointa:

$\Rightarrow (-1)^n = c$

vert. λ^{n-1} kerrointa:

$\Rightarrow (a_{11} + a_{22} + \ldots + a_{nn})(-1)^{n-1} = c(-\lambda_1 - \lambda_2 - \ldots - \lambda_n)$

$(a_{11} + a_{22} + \ldots + a_{nn})(-1)^{n-1} = (-1)^{n+1}(\lambda_1 + \lambda_2 + \ldots + \lambda_n)$

$\Rightarrow \operatorname{Tr} A = a_{11} + a_{22} + \ldots + a_{nn} = \lambda_1 + \lambda_2 + \ldots + \lambda_n$

L3)

$y' = Ay$ keskus origossa

$\Rightarrow \operatorname{Re}(\lambda_i) = 0$

$\tilde{A} = A + kI$, $k \in \mathbb{R}$

$\det(A - I\lambda) = p(\lambda)$

$\det(\tilde{A} - I\lambda) = \det(A + kI - I\lambda)$

$= \det(A - I(\lambda - k))$

$= p(\lambda - k)$

$\Rightarrow$

jos A:lla ominaisarvo λ

niin $\tilde{A}$:lla ominaisarvo $\tilde{\lambda} = \lambda + k$

$k < 0$: $\operatorname{Re}(\tilde{\lambda}_i) = \operatorname{Re}(\lambda_i + k) < 0 \Rightarrow$ origo nielu

$k > 0$: $\operatorname{Re}(\tilde{\lambda}_i) = \operatorname{Re}(\lambda_i + k) > 0 \Rightarrow$ origo lähde

systeemille $y' = \tilde{A}y$

L4) $y' = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} y + \begin{pmatrix} t \\ -3t \end{pmatrix}$

ratkaistaan ensin homog. yhtälö

$y_h' = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} y_h$

$\det(A - I\lambda) = \lambda^2 - 1 = 0 \qquad \lambda = \pm 1$

$\lambda = 1: \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} x = 0 \Rightarrow x = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\lambda = -1: \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} x = 0 \Rightarrow x = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$X = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \Rightarrow X^{-1} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

$$y_h = X e^{Dt} X^{-1} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$
$$= \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} e^t & 0 \\ 0 & e^{-t} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$
$$= \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} e^t & e^t \\ e^{-t} & -e^{-t} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$
$$= \frac{1}{2} \begin{pmatrix} e^t + e^{-t} & e^t - e^{-t} \\ e^t - e^{-t} & e^t + e^{-t} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$
$$= c_1 \begin{pmatrix} \cosh t \\ \sinh t \end{pmatrix} + c_2 \begin{pmatrix} \sinh t \\ \cosh t \end{pmatrix}$$

epähomog. erityisratk.

yrite $y_p = \begin{pmatrix} at + b \\ ct + d \end{pmatrix}$

$$\begin{pmatrix} a \\ c \end{pmatrix} = \begin{pmatrix} ct + d \\ at + b \end{pmatrix} + \begin{pmatrix} t \\ -3t \end{pmatrix}$$

$$\Rightarrow \begin{cases} a = d \\ c + 1 = 0 \\ c = b \\ a - 3 = 0 \end{cases} \Rightarrow \begin{cases} a = 3 \\ b = -1 \\ c = -1 \\ d = 3 \end{cases}$$

$$\Rightarrow y(t) = c_1 \begin{pmatrix} \cosh t \\ \sinh t \end{pmatrix} + c_2 \begin{pmatrix} \sinh t \\ \cosh t \end{pmatrix} + \begin{pmatrix} 3t - 1 \\ -t + 3 \end{pmatrix}$$

L5) $$y' = \begin{pmatrix} 3 & -4 \\ 1 & -2 \end{pmatrix} y + \begin{pmatrix} 20\cos t \\ 0 \end{pmatrix}, \quad y(0) = \begin{pmatrix} 0 \\ 8 \end{pmatrix}$$

homog. yhtälön ratkaisu:

$$y_h' = \begin{pmatrix} 3 & -4 \\ 1 & -2 \end{pmatrix} y_h$$

$$\det(A - I\lambda) = (3-\lambda)(-2-\lambda) + 4 = 0$$

$$\lambda^2 - \lambda - 2 = 0 \qquad \lambda_1 = -1, \quad \lambda_2 = 2$$

$$\lambda = -1: \begin{pmatrix} 4 & -4 \\ 1 & -1 \end{pmatrix} x = 0 \Rightarrow x = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\lambda = 2: \begin{pmatrix} 1 & -4 \\ 1 & -4 \end{pmatrix} x = 0 \Rightarrow x = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

$$\Rightarrow y_h(t) = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 4 \\ 1 \end{pmatrix} e^{2t}$$

epähomog. erityisratk:

yrite: $y_p = \begin{pmatrix} a\cos t + b\sin t \\ c\cos t + d\sin t \end{pmatrix}$

sijoitetaan

$$\begin{pmatrix} -a\sin t + b\cos t \\ -c\sin t + d\cos t \end{pmatrix} =$$

$$\begin{pmatrix} 3(a\cos t + b\sin t) - 4(c\cos t + d\sin t) \\ a\cos t + b\sin t - 2(c\cos t + d\sin t) \end{pmatrix} + \begin{pmatrix} 20\cos t \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{cases} -a = 3b - 4d \\ b = 3a - 4c + 20 \\ -c = b - 2d \\ d = a - 2c \end{cases} \Rightarrow \begin{pmatrix} 1 & 3 & 0 & -4 \\ 3 & -1 & -4 & 0 \\ 0 & 1 & 1 & -2 \\ 1 & 0 & -2 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 0 \\ -20 \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{cases} a = -14 \\ b = 2 \\ c = -6 \\ d = -2 \end{cases}$$

$y = y_h + y_p$

$$y = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 4 \\ 1 \end{pmatrix} e^{2t} + \begin{pmatrix} -14\cos t + 2\sin t \\ -6\cos t - 2\sin t \end{pmatrix}$$

alkuehto

$$y(0) = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 4 \\ 1 \end{pmatrix} + \begin{pmatrix} -14 \\ -6 \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \end{pmatrix}$$

$$\Rightarrow \begin{cases} c_1 + 4c_2 - 14 = 0 \\ c_1 + c_2 - 6 = 8 \end{cases} \Rightarrow \begin{cases} c_1 + 4c_2 = 14 \\ c_1 + c_2 = 14 \end{cases}$$

$$3c_2 = 0 \qquad \Rightarrow \quad c_1 = 14$$

$$\Rightarrow \; y(t) = 14 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + \begin{pmatrix} -14\cos t + 2\sin t \\ -6\cos t - 2\sin t \end{pmatrix}$$

L6) $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}' = \begin{pmatrix} -y_1 + y_2 + y_2^2 \\ -y_1 - y_2 \end{pmatrix} =: \begin{pmatrix} f(y_1, y_2) \\ g(y_1, y_2) \end{pmatrix}$

kriittiset pisteet $\begin{cases} -y_1 + y_2 + y_2^2 = 0 \\ -y_1 - y_2 = 0 \end{cases}$

$\Rightarrow -y_1 = y_2 \Rightarrow 2y_2 + y_2^2 = 0$

$y_2(2 + y_2) = 0$

$y_2 = 0$ tai $y_2 = -2$

$(y_1, y_2) = (0,0)$ tai $(y_1, y_2) = (2, -2)$

$(y_1, y_2) = (0,0)$:

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}' = \begin{pmatrix} f_{y_1}(0,0) & f_{y_2}(0,0) \\ g_{y_1}(0,0) & g_{y_2}(0,0) \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}' = \begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$\det(A - I\lambda) = (-1-\lambda)^2 + 1 = 0$

$-1-\lambda = \pm i \qquad \lambda = -1 \pm i$

$\mathrm{Re}(\lambda_i) < 0 \Rightarrow (0,0)$ spiraalinielu

$(y_1, y_2) = (2, -2)$

$\tilde{y}_1 = y_1 - 2$

$\tilde{y}_2 = y_2 + 2$

$$\begin{pmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{pmatrix}' = \begin{pmatrix} f_{y_1}(2,-2) & f_{y_2}(2,-2) \\ g_{y_1}(2,-2) & g_{y_2}(2,-2) \end{pmatrix} \begin{pmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{pmatrix}$$

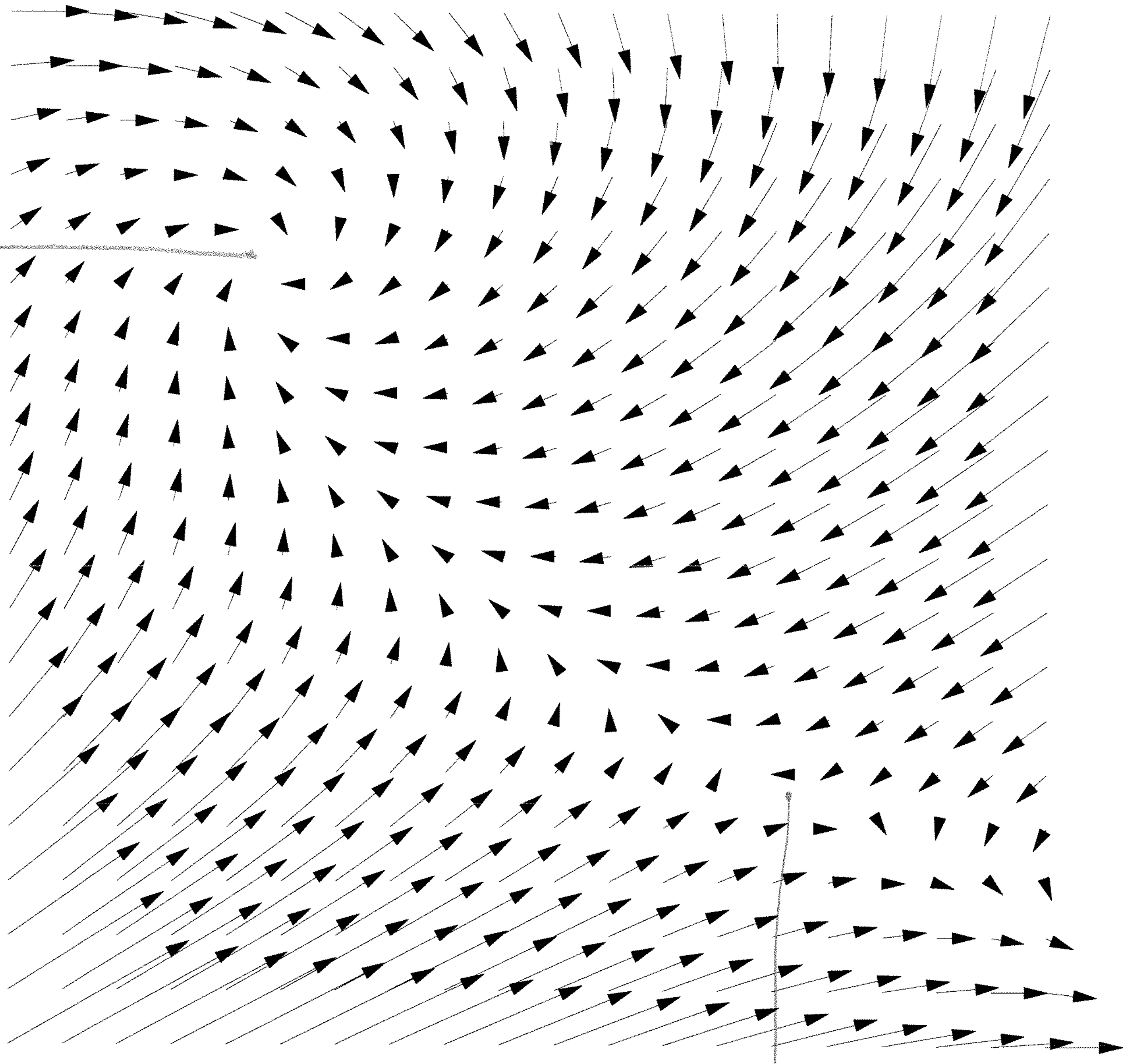
$\begin{pmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{pmatrix}' = \begin{pmatrix} -1 & -3 \\ -1 & -1 \end{pmatrix}$ $\quad \det(A - I\lambda) = (-1-\lambda)^2 - 3 = 0$

$-1-\lambda = \pm\sqrt{3} \quad \lambda = -1 \pm \sqrt{3}$

om.arvot eri merkkiset $\Rightarrow (2,-2)$ satula

```
In[20]:= << Graphics`PlotField`
```

```
In[37]:= PlotVectorField[{-y1 + y2 + y2^2, -y1 - y2}, {y1, -1, 3}, {y2, -3, 1},
          PlotPoints → 20, ScaleFunction → (0.1 # &), ScaleFactor → None]
```

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Out[37]= - Graphics -
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