Mallivastaukset C3-ii
Vko 47

A1)

$$A=\begin{bmatrix}0,6 & 0,4 & 0\\ 0,1 & 0,1 & 0,8\\ 0,2 & 0,4 & 0,4\end{bmatrix}$$

Markovin prosessissa seuraava tila saadaan laskemalla $y=A^Tx$, missä x on edellinen tila. Raja on saavutettu kun $A^Tx=x$.
(A:lla tulee olla ominaisarvo 1, mikä toteutuu tässä tapauksessa. A:n ominaisarvot ovat $1; 0,4; -0,3$)
$A^Tx=x$:n epätriviaali ratkaisu on se A^T:n ominaisvektori, joka vastaa ominaisarvoa 1. Saadaan:

$$(A^T-I)x=0$$

$$\Rightarrow \begin{bmatrix}-0,4 & 0,1 & 0,2\\ 0,4 & -0,9 & 0,4\\ 0 & 0,8 & -0,6\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=0$$

Kerrotaan riveittäin 10:llä ja Gauss-Jordanoidaan:

$$\Rightarrow \begin{bmatrix}-4 & 1 & 2\\ 4 & -9 & 4\\ 0 & 8 & -6\end{bmatrix} \Rightarrow \begin{bmatrix}-4 & 1 & 2\\ 0 & -8 & 6\\ 0 & 8 & -6\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}-4 & 1 & 2\\ 0 & -8 & 6\\ 0 & 0 & 0\end{bmatrix} \Rightarrow \begin{cases}-4x_1+x_2+2x_3=0 & (2)\\ -8x_2+6x_3=0 & (1)\end{cases}$$

$$(1)\Rightarrow x_2=\frac{3}{4}x_3 \qquad (2)\Rightarrow -4x_1+\frac{3}{4}x_3+2x_3=0$$

$$\Rightarrow x_1=\frac{11}{16}x_3$$

Haluamme sen om.vektorin, jonka komponenttien summa on 1:

$$\Rightarrow \frac{11}{16}x_3+\frac{3}{4}x_3+x_3=1 \Rightarrow \underline{x_1=\frac{11}{39}\approx 0,28}$$

$$\underline{x_2=\frac{4}{13}\approx 0,31} \quad \text{ja} \quad \underline{x_3=\frac{16}{39}\approx 0,41}$$

A2)

$$H(\varphi)=\begin{pmatrix}\cos 2\varphi & \sin 2\varphi \\ \sin 2\varphi & -\cos 2\varphi\end{pmatrix} \qquad x=\begin{pmatrix}\cos\alpha \\ \sin\alpha\end{pmatrix}$$

$$H^2=\begin{pmatrix}\cos 2\varphi & \sin 2\varphi \\ \sin 2\varphi & -\cos 2\varphi\end{pmatrix}\begin{pmatrix}\cos 2\varphi & \sin 2\varphi \\ \sin 2\varphi & -\cos 2\varphi\end{pmatrix}$$

$$=\begin{pmatrix}\cos^2 2\varphi+\sin^2 2\varphi & \sin 2\varphi\cos 2\varphi-\sin 2\varphi\cos 2\varphi \\ \sin 2\varphi\cos 2\varphi-\sin 2\varphi\cos 2\varphi & \sin^2 2\varphi+\cos^2 2\varphi\end{pmatrix}$$

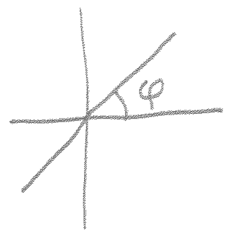
$$=\begin{pmatrix}1 & 0 \\ 0 & 1\end{pmatrix} \Rightarrow H \text{ on ortogonaalinen.}$$

$$y=Hx=\begin{pmatrix}\cos 2\varphi & \sin 2\varphi \\ \sin 2\varphi & -\cos 2\varphi\end{pmatrix}\begin{pmatrix}\cos\alpha \\ \sin\alpha\end{pmatrix}$$

$$=\begin{pmatrix}\cos\alpha\cos 2\varphi+\sin\alpha\sin 2\varphi \\ \cos\alpha\sin 2\varphi-\sin\alpha\cos 2\varphi\end{pmatrix}$$

Valitoaan $\varphi=\frac{\pi}{2}$: $Hx=\begin{pmatrix}-\cos\alpha \\ \sin\alpha\end{pmatrix}$

$\varphi=\pi$: $Hx=\begin{pmatrix}\cos\alpha \\ -\sin\alpha\end{pmatrix}$



$\varphi=\frac{\pi}{4}$: $Hx=\begin{pmatrix}\sin\alpha \\ \cos\alpha\end{pmatrix}$

Eli H on peilaus suoran, joka on φ:n kulmassa x-akseliin nähden, suhteen.

$$\mathrm{Det}(H)=\begin{vmatrix}\cos 2\varphi & \sin 2\varphi \\ \sin 2\varphi & -\cos 2\varphi\end{vmatrix}=-(\cos^2 2\varphi+\sin^2 2\varphi)$$
$$=-1 \quad \text{vrt. } \mathrm{Det}(K)=1$$

Huomataan, että $\mathrm{Det}(H)=-1$ kun taas $\mathrm{Det}(K)$ olisi 1. Peilausmatriiseilla determinantti on -1 ja kääntömatriiseilla 1.

A3)

$$Q = 4x_1^2 + 2\sqrt{3}\,x_1x_2 + 2x_2^2 = 10$$

$$Q = x^T A x$$

$$\Rightarrow A = \begin{bmatrix} 4 & \sqrt{3} \\ \sqrt{3} & 2 \end{bmatrix}$$

A:n ominaisarvot:

$$\begin{vmatrix} 4-\lambda & \sqrt{3} \\ \sqrt{3} & 2-\lambda \end{vmatrix} = (4-\lambda)(2-\lambda) - (\sqrt{3})^2 = \lambda^2 - 6\lambda + 5$$

$$\lambda^2 - 6\lambda + 5 = 0 \Rightarrow \lambda_1 = 1 \quad \lambda_2 = 5$$

Nyt $Q = y_1^2 + 5y_2^2 = 10 \Rightarrow \frac{y_1^2}{10} + \frac{y_2^2}{5} = 1$,

mikä on ellipsi.

A:n ominaisvektorit:

$$\lambda_1 = 1: \quad (A-I)x = 0 \Rightarrow \begin{cases} 3x_1 + \sqrt{3}\,x_3 = 0 \\ \sqrt{3}\,x_1 + x_2 = 0 \end{cases}$$

$$\Rightarrow x_2 = -\sqrt{3}\,x_1 \quad \text{Val. } x_1 = 1$$

$$\Rightarrow x = \begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix} \quad |x| = \sqrt{1^2 + (-\sqrt{3})^2} = \frac{1}{2}$$

$$\Rightarrow x_{N1} = \begin{bmatrix} \frac{1}{2} \\ -\sqrt{3}/2 \end{bmatrix} \text{(normeerattu)}$$

$$\lambda_2 = 5: \quad (A-I)x = 0 \Rightarrow \begin{cases} -x_1 + \sqrt{3}\,x_2 = 0 \\ \sqrt{3}\,x_1 - 3x_2 = 0 \end{cases}$$

$$\Rightarrow x_1 = \sqrt{3}\,x_2 \quad \text{Val. } x_2 = 1$$

$$\Rightarrow x = \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix} \quad |x| = \frac{1}{2}$$

$$x_{N2} = \begin{bmatrix} \sqrt{3}/2 \\ 1/2 \end{bmatrix}$$

$$\Rightarrow X = [x_{N1} \ \ x_{N2}]$$

$$x = Xy = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} y$$

L1)

$$A = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^T = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$AA^T = \begin{pmatrix} \cos^2\theta + \sin^2\theta & \cos\theta\sin\theta - \cos\theta\sin\theta & 0 \\ \cos\theta\sin\theta - \cos\theta\sin\theta & \sin^2\theta + \cos^2\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\Rightarrow$ A on ortogonaalinen. Lasketaan A:n ominaisarvot

$$\det(A - \lambda I) = \begin{vmatrix} \cos\theta - \lambda & -\sin\theta & 0 \\ \sin\theta & \cos\theta - \lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix}$$

$$= (\cos\theta - \lambda)(\cos\theta - \lambda)(1-\lambda) + 0 + 0 - 0 - 0 - (1-\lambda)(-\sin\theta)(\sin\theta)$$

$$= (\cos\theta - \lambda)(\cos\theta - \lambda)(1-\lambda) + \sin^2\theta(1-\lambda)$$

$$= \ldots = -\lambda^3 + \lambda^2(1 + 2\cos\theta) - \lambda(2\cos\theta)$$

$$\Rightarrow \lambda_1 = 1, \quad \lambda_2 = \cos\theta - i\sin\theta, \quad \lambda_3 = \cos\theta + i\sin\theta$$

$$\operatorname{Det}(A) = \begin{vmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$$

$\Rightarrow$ A on kiertomatriisi. $y = Ax$

$$x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow y = \begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix} \qquad x = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \Rightarrow y = \begin{bmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{bmatrix}$$

$$x = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow y = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$\Rightarrow$ A on kierto z-akselin suhteen.

L2)

$$A=\begin{pmatrix}2 & 7\\ 6 & -9\end{pmatrix}$$

$$\det(A-\lambda I)=\begin{vmatrix}2-\lambda & 7\\ 6 & -9-\lambda\end{vmatrix}=(2-\lambda)(-9-\lambda)-42$$
$$=-18-2\lambda+9\lambda+\lambda^2-42$$

$$=\lambda^2+7\lambda-60 \Rightarrow \lambda_1=-12 \quad \lambda_2=5$$

λ_1: $(A+12I)x=0$

$$\Rightarrow \begin{pmatrix}14 & 7\\ 6 & 3\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}=0 \Rightarrow \begin{cases}14x_1+7x_2=0\\ 6x_1+3x_2=0\end{cases}$$

$$\Rightarrow x_2=-2x_1 \quad \text{Val. } x_1=1 \Rightarrow x_2=-2 \Rightarrow x=\begin{pmatrix}1\\ -2\end{pmatrix}$$

λ_2: $(A-5I)x=0$

$$\Rightarrow \begin{pmatrix}-3 & 7\\ 6 & -14\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}=0 \Rightarrow \begin{cases}-3x_1+7x_2=0\\ 6x_1-14x_2=0\end{cases}$$

$$\Rightarrow 3x_1=7x_2 \Rightarrow x_1=\tfrac{7}{3}x_2 \quad \text{Val. } x_2=3 \Rightarrow x_1=7 \Rightarrow x=\begin{pmatrix}7\\ 3\end{pmatrix}$$

$$\Rightarrow X=\underline{\begin{pmatrix}1 & 7\\ -2 & 3\end{pmatrix}}$$ Ratkaistaan X^{-1}:

$$\left(\begin{array}{cc|cc}1 & 7 & 1 & 0\\ -2 & 3 & 0 & 1\end{array}\right)\cdot 2 \Rightarrow \left(\begin{array}{cc|cc}1 & 7 & 1 & 0\\ 0 & 17 & 2 & 1\end{array}\right):17$$

$$\Rightarrow \left(\begin{array}{cc|cc}1 & 7 & 1 & 0\\ 0 & 1 & 2/17 & 1/17\end{array}\right)\cdot(-7) \Rightarrow \left(\begin{array}{cc|cc}1 & 0 & 3/17 & -7/17\\ 0 & 1 & 2/17 & 1/17\end{array}\right)$$

$$\Rightarrow D=X^{-1}AX=\frac{1}{17}\begin{pmatrix}3 & -7\\ 2 & 1\end{pmatrix}\begin{pmatrix}2 & 7\\ 6 & -9\end{pmatrix}\begin{pmatrix}1 & 7\\ -2 & 3\end{pmatrix}$$

$$=\frac{1}{17}\begin{pmatrix}3 & -7\\ 2 & 1\end{pmatrix}\begin{pmatrix}2-14 & 14+21\\ 6+18 & 42-27\end{pmatrix}=\frac{1}{17}\begin{pmatrix}3 & -7\\ 2 & 1\end{pmatrix}\begin{pmatrix}-12 & 35\\ 24 & 15\end{pmatrix}$$

$$=\frac{1}{17}\begin{pmatrix}-36-168 & 105-105\\ -24+24 & 70+15\end{pmatrix}=\frac{1}{17}\begin{pmatrix}-204 & 0\\ 0 & 85\end{pmatrix}$$

$$=\underline{\begin{pmatrix}-12 & 0\\ 0 & 5\end{pmatrix}}$$

L3) $A=\begin{pmatrix} 3 & 10 & -15 \\ -18 & 39 & 9 \\ -24 & 40 & -15 \end{pmatrix}$

$$\det(A-\lambda I)=\begin{vmatrix} 3-\lambda & 10 & -15 \\ -18 & 39-\lambda & 9 \\ -24 & 40 & -15-\lambda \end{vmatrix} = \dots = -\lambda^3+27\lambda^2+1053\lambda-10935$$

$\Rightarrow \lambda_1=45,\ \lambda_2=9,\ \lambda_3=-27$

λ_1: $(A-45I)x=0$

$$\Rightarrow \begin{pmatrix} -42 & 10 & -15 \\ -18 & -6 & 9 \\ -24 & 40 & -60 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=0$$

$$\begin{pmatrix} -42 & 10 & -15 \\ -18 & -6 & 9 \\ -24 & 40 & -60 \end{pmatrix} \cdot -\tfrac{3}{7} \quad \cdot -\tfrac{4}{7}$$

$$\Rightarrow \begin{pmatrix} -42 & 10 & -15 \\ 0 & -72/7 & 108/7 \\ 0 & 240/7 & -360/7 \end{pmatrix} \cdot \tfrac{10}{3} \Rightarrow \begin{pmatrix} -42 & 10 & -15 \\ 0 & -72/7 & 108/7 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow \begin{cases} -42x_1+10x_2-15x_3=0 \\ -72x_2+108x_3=0 \Rightarrow x_2=\tfrac{3}{2}x_3 \end{cases}$$

Val. $x_3=2 \Rightarrow x_2=3$

$\Rightarrow -42x_1+30-30=0 \Rightarrow x_1=0$

$$\Rightarrow x=\begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}$$

λ_2: $(A-9I)x=0$

$$\Rightarrow \begin{cases} -6x_1+10x_2-15x_3=0 \\ -18x_1+30x_2+9x_3=0 \\ -24x_1+40x_2-24x_3=0 \end{cases}$$

$\Rightarrow x_3=0,\ x_1=\frac{5}{3}x_2$ Val. $x_2=3 \Rightarrow x_1=5$

$$\Rightarrow x=\begin{bmatrix} 5 \\ 3 \\ 0 \end{bmatrix}$$

L3 jatkuu

$\lambda_3: (A+27I)x = 0$

$$\Rightarrow \begin{cases} 30x_1 + 10x_2 - 15x_3 = 0 \\ -18x_1 + 66x_2 + 9x_3 = 0 \\ -24x_1 + 40x_2 + 12x_3 = 0 \end{cases}$$

$$\Rightarrow x_1 = 1,\ x_2 = 0,\ x_3 = 2$$

$$\Rightarrow x = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

$$\Rightarrow X = \begin{pmatrix} 5 & 0 & 1 \\ 3 & 3 & 0 \\ 0 & 2 & 2 \end{pmatrix}$$ Ratkaistaan X^{-1}:

$$\left(\begin{array}{ccc|ccc} 5 & 0 & 1 & 1 & 0 & 0 \\ 3 & 3 & 0 & 0 & 1 & 0 \\ 0 & 2 & 2 & 0 & 0 & 1 \end{array}\right) -\tfrac{3}{5} \Rightarrow \left(\begin{array}{ccc|ccc} 5 & 0 & 1 & 1 & 0 & 0 \\ 0 & 3 & -3/5 & -3/5 & 1 & 0 \\ 0 & 2 & 2 & 0 & 0 & 1 \end{array}\right) -\tfrac{2}{3}$$

$$\Rightarrow \left(\begin{array}{ccc|ccc} 5 & 0 & 1 & 1 & 0 & 0 \\ 0 & 3 & -3/5 & -3/5 & 1 & 0 \\ 0 & 0 & 12/5 & 2/5 & -2/3 & 1 \end{array}\right) \begin{array}{c} \\ \cdot 5 \\ \cdot 5 \end{array} \Rightarrow \left(\begin{array}{ccc|ccc} 5 & 0 & 1 & 1 & 0 & 0 \\ 0 & 15 & -3 & -3 & 5 & 0 \\ 0 & 0 & 12 & 2 & -10/3 & 5 \end{array}\right) \tfrac{1}{4}\ -\tfrac{1}{12}$$

$$\Rightarrow \left(\begin{array}{ccc|ccc} 5 & 0 & 0 & 5/6 & 5/18 & -5/12 \\ 0 & 15 & 0 & -5/2 & 50/12 & 5/4 \\ 0 & 0 & 12 & 2 & -10/3 & 5 \end{array}\right) \begin{array}{c} :5 \\ :15 \\ :12 \end{array}$$

$$\Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/6 & 1/18 & -1/12 \\ 0 & 1 & 0 & -1/6 & 5/18 & 1/12 \\ 0 & 0 & 1 & 1/6 & -5/18 & 5/12 \end{array}\right) \Rightarrow X^{-1} = \frac{1}{18}\begin{pmatrix} 3 & 1 & -3/2 \\ -3 & 5 & 3/2 \\ 3 & -5 & 15/2 \end{pmatrix}$$

$$D = X^{-1}AX = X^{-1}\begin{pmatrix} 3 & 10 & -15 \\ -18 & 39 & 9 \\ -24 & 40 & -15 \end{pmatrix}\begin{pmatrix} 5 & 0 & 1 \\ 3 & 3 & 0 \\ 0 & 2 & 2 \end{pmatrix}$$

$$= X^{-1}\begin{pmatrix} 45 & 0 & -27 \\ 27 & 135 & 0 \\ 0 & 90 & -54 \end{pmatrix} = \frac{1}{18}\begin{pmatrix} 3 & 1 & -3/2 \\ -3 & 5 & 3/2 \\ 3 & -5 & 15/2 \end{pmatrix}\begin{pmatrix} 45 & 0 & -27 \\ 27 & 135 & 0 \\ 0 & 90 & -54 \end{pmatrix}$$

$$= \frac{1}{18}\begin{pmatrix} 162 & 0 & 0 \\ 0 & 810 & 0 \\ 0 & 0 & -486 \end{pmatrix} = \begin{pmatrix} 9 & 0 & 0 \\ 0 & 45 & 0 \\ 0 & 0 & -27 \end{pmatrix}$$

4

B) a) $V = \{X \in \mathbb{R}^{3\times 3} \mid X^T = X\}$

$A, B \in V \Rightarrow (\alpha A + \beta B)^T = \alpha A^T + \beta B^T = \alpha A + \beta B \quad \forall \alpha, \beta \in \mathbb{R}$

$\alpha A + \beta B$ symmetrinen $\Rightarrow \alpha A + \beta B \in V \quad \forall \alpha, \beta \in \mathbb{R}$

V on vektoriavaruus

kanta esim. $\left\{ \begin{bmatrix} 1&0&0\\0&0&0\\0&0&0 \end{bmatrix}, \begin{bmatrix} 0&1&0\\1&0&0\\0&0&0 \end{bmatrix}, \begin{bmatrix} 0&0&1\\0&0&0\\1&0&0 \end{bmatrix}, \begin{bmatrix} 0&0&0\\0&1&0\\0&0&0 \end{bmatrix}, \begin{bmatrix} 0&0&0\\0&0&1\\0&1&0 \end{bmatrix}, \begin{bmatrix} 0&0&0\\0&0&0\\0&0&1 \end{bmatrix} \right\}$

$\dim V = 6$ [itse asiassa $\dim V \geq 6$, mutta loppu voidaan perustella

$U := \operatorname{span}\left\{ \begin{bmatrix} 0&0&0\\1&0&0\\0&0&0 \end{bmatrix}, \begin{bmatrix} 0&0&0\\0&0&0\\1&0&0 \end{bmatrix}, \begin{bmatrix} 0&0&0\\0&0&0\\0&1&0 \end{bmatrix} \right\}$, $U \cap V = \{0\}$, $\dim U = 3$

$\dim U + \dim V \leq \underbrace{9}_{\dim \mathbb{R}^{3\times 3}} \Rightarrow \dim V \leq 6$]

b) $W = \left\{ \begin{bmatrix} v_1\\v_2\\v_3 \end{bmatrix} \in \mathbb{R}^3 \mid v_1 - 2v_2 + v_3 = 0 \right\} = \left\{ \begin{bmatrix} v_1\\v_2\\2v_2 - v_1 \end{bmatrix} \in \mathbb{R}^3 \right\}$

$= \left\{ v_1 \begin{bmatrix} 1\\0\\-1 \end{bmatrix} + v_2 \begin{bmatrix} 0\\1\\2 \end{bmatrix} \mid v_1, v_2 \in \mathbb{R} \right\}$

$= \operatorname{span}\left\{ \begin{bmatrix} 1\\0\\-1 \end{bmatrix}, \begin{bmatrix} 0\\1\\2 \end{bmatrix} \right\}$

on vektoriavaruus, dimensio 2, kanta $\left\{ \begin{bmatrix} 1\\0\\-1 \end{bmatrix}, \begin{bmatrix} 0\\1\\2 \end{bmatrix} \right\}$

L5)

$A = \begin{bmatrix} 8 & 0 & 3 \\ 2 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \quad T = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \hat{A} = T^{-1}AT, \quad T^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = T$

$$\hat{A} = T \begin{bmatrix} 0 & 8 & 3 \\ 2 & 2 & 1 \\ 0 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 1 \\ 0 & 8 & 3 \\ 0 & 2 & 3 \end{bmatrix}$$

ominaisarvot $\hat{A}$:lle

$$\begin{vmatrix} 2-\hat{\lambda} & 2 & 1 \\ 0 & 8-\hat{\lambda} & 3 \\ 0 & 2 & 3-\hat{\lambda} \end{vmatrix} = (2-\hat{\lambda}) \begin{vmatrix} 8-\hat{\lambda} & 3 \\ 2 & 3-\hat{\lambda} \end{vmatrix} = (2-\hat{\lambda})\left[(8-\hat{\lambda})(3-\hat{\lambda}) - 6\right] = 0$$

$$(2-\hat{\lambda})(\hat{\lambda}^2 - 11\hat{\lambda} + 18) = 0 \Rightarrow (2-\hat{\lambda})(\hat{\lambda}-2)(\hat{\lambda}-9) = 0$$

ominaisarvot $\underline{\hat{\lambda}_{1,2} = 2, \quad \hat{\lambda}_3 = 9}$

A:lle

$$\begin{vmatrix} 8-\lambda & 0 & 3 \\ 2 & 2-\lambda & 1 \\ 2 & 0 & 3-\lambda \end{vmatrix} = (2-\lambda) \begin{vmatrix} 8-\lambda & 3 \\ 2 & 3-\lambda \end{vmatrix} = (2-\lambda)(\lambda-2)(\lambda-9) = 0$$

$\underline{\lambda_{1,2} = 2, \quad \lambda_3 = 9}$ samat ominaisarvot

$\hat{A}$:n ominaisvektorit

$$\hat{\lambda} = 2: \begin{bmatrix} 0 & 2 & 1 \\ 0 & 6 & 3 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \Leftrightarrow \begin{bmatrix} 0 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \Leftrightarrow z = -2y$$

$$t \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + s \cdot \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}, \quad t, s \in \mathbb{R}$$

$$\underline{y_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}} \quad \underline{y_2 = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}}, \quad Ty_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad Ty_2 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

$$ATy_1 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} = 2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 2Ty_1, \quad ATy_2 = \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} = 2Ty_2$$

$$\hat{\lambda} = 9: \begin{bmatrix} -7 & 2 & 1 \\ 0 & -1 & 3 \\ 0 & 2 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \Leftrightarrow \begin{bmatrix} -7 & 0 & 7 \\ 0 & -1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \Leftrightarrow \begin{cases} -x + z = 0 \\ -y + 3z = 0 \\ 0 = 0 \end{cases}$$

$$x = z, \quad y = 3z \Rightarrow \underline{y_3 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}} \qquad Ty_3 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

$$ATy_3 = A \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 27 \\ 9 \\ 9 \end{bmatrix} = 9 \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} = 9\,Ty_3$$

Siispä Ty_i on A:n ominaisvektori kaikilla $\tilde{A}$:n ominaisvektoreilla y_i

2(c)

a) $u = \begin{bmatrix} 4 \\ -3 \\ -1 \\ 5 \\ -5 \end{bmatrix}$

$\|u\|_1 = |4| + |-3| + |-1| + |5| + |-5| = 18$

$\|u\|_2 = \sqrt{4^2 + (-3)^2 + (-1)^2 + 5^2 + (-5)^2} = \sqrt{76} = 2\sqrt{19}$

$\|u\|_\infty = \max|u_k| = 5$

$V = \begin{bmatrix} 5 \\ 4 \\ 3 \\ 3 \\ 2 \\ 1 \end{bmatrix}$

$\|v\|_1 = |5| + |4| + |3| + |3| + |2| + |1| = 18$

$\|v\|_2 = \sqrt{5^2 + 4^2 + 3^2 + 3^2 + 2^2 + 1^2} = \sqrt{64} = 8$

$\|v\|_\infty = \max|v_k| = 5$

b) $\|x\|_\infty = \max\limits_{1 \le k \le n} |x_k| \le \sum\limits_{j=1}^{n} |x_j| = \|x\|_1 \le \sum\limits_{j=1}^{n} \max\limits_{1 \le k \le n} |x_k| = n \cdot \max\limits_{1 \le k \le n} |x_k|$

$= n \cdot \|x\|_\infty$

$\Rightarrow \|x\|_\infty \le \|x\|_1 \le n \cdot \|x\|_\infty$

tästä heti $\|x\|_\infty \le \|x\|_1$ ja $\frac{1}{n}\|x\|_1 \le \frac{1}{n} \cdot n\|x\|_\infty = \|x\|_\infty$

$\Rightarrow \frac{1}{n}\|x\|_1 \le \|x\|_\infty \le \|x\|_1$