

L1

VKO 44 / C3-II / SN

$$A = \begin{pmatrix} 6 & -2 & -2 \\ 10 & -3 & 1 \\ -10 & 5 & 1 \end{pmatrix}$$

$$a = \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix}$$

$$d = (3 \ 0 \ 8)$$

$$B = \begin{pmatrix} 9 & 4 & -4 \\ 4 & 7 & 0 \\ -4 & 0 & 11 \end{pmatrix}$$

(a) $a^T a = \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix} (5 \ 1 \ 2) = 5^2 + 1^2 + 2^2 = \underline{\underline{30}}$

"EXTRA" $a^T A = \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix} A = \begin{pmatrix} 30 + 10 - 20 \\ -10 - 3 + 10 \\ -10 + 1 + 2 \end{pmatrix}^T = (20 \ -3 \ -7)$

(b) $d A = \begin{pmatrix} 3 \cdot 6 & -8 \cdot 10 \\ -3 \cdot 2 & +8 \cdot 5 \\ -3 \cdot 2 & +8 \cdot 1 \end{pmatrix}^T = \underline{\underline{(-62 \ 34 \ 2)}}$

(c) $5B(3a + 2d)$ ei ole määritetty,
koska $3a$ ja $2d$ ovat eri kokoa.

"EXTRA": $5B(3a + 2d^T) = 5B(21 \ 3 \ 22)^T$
 $= 5 \begin{pmatrix} 9 & 4 & -4 \\ 4 & 7 & 0 \\ -4 & 0 & 11 \end{pmatrix} \begin{pmatrix} 21 \\ 3 \\ 22 \end{pmatrix} = 5 \begin{pmatrix} 9 \cdot 21 + 4 \cdot 3 - 4 \cdot 22 \\ 4 \cdot 21 + 7 \cdot 3 \\ -4 \cdot 21 + 11 \cdot 22 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 565 \\ 525 \\ 790 \end{pmatrix}}}$

(d) $15B a + 10B d^T = 5B(3a + 2d^T)$
 $=$ sama kuin "c-EXTRA"

L 2

$$\begin{aligned}
 a) \quad (A+B)^2 &= \begin{pmatrix} 15 & 2 & -6 \\ 14 & 4 & 1 \\ -14 & 5 & 12 \end{pmatrix} \begin{pmatrix} 15 & 2 & -6 \\ 14 & 4 & 1 \\ -14 & 5 & 12 \end{pmatrix} \\
 &= \begin{pmatrix} 15 \cdot 15 + 2 \cdot 14 + 6 \cdot 14 & 15 \cdot 2 + 2 \cdot 4 - 6 \cdot 5 & -15 \cdot 6 + 2 \cdot 12 \\ 14 \cdot 15 + 4 \cdot 14 - 14 & 14 \cdot 2 + 4 \cdot 4 + 1 \cdot 5 & -14 \cdot 6 + 4 \cdot 1 + 1 \cdot 12 \\ -14 \cdot 15 + 5 \cdot 14 - 12 \cdot 14 & -14 \cdot 2 + 5 \cdot 4 + 12 & +14 \cdot 6 + 5 + 12 \cdot 12 \end{pmatrix} \\
 &= \begin{pmatrix} 337 & 8 & -160 \\ 252 & 49 & -68 \\ -308 & 52 & 233 \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad A^2 + AB + BA + B^2 \\
 &= A(A+B) + B(A+B) = (A+B)(A+B) \\
 &= (A+B)^2 = \underline{\underline{\text{sama kuin L 2 a)}}.
 \end{aligned}$$

$$\begin{aligned}
 c) \quad A^2 + 2AB + B^2 \\
 &= \begin{pmatrix} 36 & -16 & -16 \\ 20 & -6 & -22 \\ -20 & 10 & 26 \end{pmatrix} + 2 \begin{pmatrix} 54 & 10 & -46 \\ 74 & 19 & -29 \\ -74 & -5 & 51 \end{pmatrix} + \begin{pmatrix} 113 & 64 & -80 \\ 64 & 65 & -16 \\ -80 & -16 & 137 \end{pmatrix} \\
 &= \begin{pmatrix} 257 & 68 & -188 \\ 232 & 97 & -96 \\ -248 & -16 & 265 \end{pmatrix} \neq (A+B)^2
 \end{aligned}$$

L3

VKO 44 / CS-II / SN

$$\left(\begin{array}{ccc|c} 14 & -2 & -4 & 0 \\ 18 & -2 & -6 & 0 \\ 4 & 8 & -14 & 0 \end{array} \right) \begin{array}{l} +4 \\ \\ - \end{array} \quad \begin{cases} 14x - 2y - 4z = 0 \\ 18x - 2y - 6z = 0 \\ 4x + 8y - 14z = 0 \end{cases}$$

$$\left(\begin{array}{ccc|c} 14 & -2 & -4 & 0 \\ 18 & -2 & -6 & 0 \\ 60 & 0 & -30 & 0 \end{array} \right) \quad | :30$$

$$\left(\begin{array}{ccc|c} 7 & -1 & -2 & 0 \\ 9 & -1 & -3 & 0 \\ 2 & 0 & -1 & 0 \end{array} \right) \begin{array}{l} \\ -1 \\ -1 \end{array}$$

$$\left(\begin{array}{ccc|c} 7 & -1 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 0 & -1 & 0 \end{array} \right)$$

$$\text{Eli: } \begin{cases} 7x - y - 2z = 0 \\ 2x = z \end{cases}$$

$$\begin{cases} z = 2x \\ y = 7x - 2z = 3x \end{cases}$$

(x on vapaa)

L4

Vko 44/C3-II/SN

$$A = \begin{pmatrix} 0 & 3 & 4 \\ -3 & 0 & 5 \\ -4 & 5 & 0 \end{pmatrix}$$

$$\begin{aligned} \det A &= 0 \begin{vmatrix} 0 & 5 \\ 5 & 0 \end{vmatrix} - 3 \begin{vmatrix} -3 & 5 \\ -4 & 0 \end{vmatrix} + 4 \begin{vmatrix} -3 & 0 \\ -4 & 5 \end{vmatrix} \\ &= -3(0 - (-20)) + 4(-15 - 0) \\ &= -60 - 60 = -120 \end{aligned}$$

$$\Rightarrow \underline{\underline{\text{Rank } A = 3}}$$

A:n rivit ja sarakevektorit virittävät avaruuden.

"EXTRA": Jos

$$A = \begin{pmatrix} 0 & 3 & 4 \\ -3 & 0 & -5 \\ -4 & 5 & 0 \end{pmatrix} :$$

$$\begin{pmatrix} 0 & 3 & 4 \\ -3 & 0 & -5 \\ -4 & 5 & 0 \end{pmatrix} \begin{array}{l} 1 \cdot 4 \\ 1 \cdot 3 \end{array}$$

$$\begin{pmatrix} 0 & 3 & 4 \\ -12 & 0 & -20 \\ -12 & 15 & 0 \end{pmatrix} \begin{array}{l} \uparrow \\ \leftarrow \end{array} -1$$

$$\begin{pmatrix} 0 & 3 & 4 \\ -12 & 0 & -20 \\ 0 & 15 & 20 \end{pmatrix} \begin{array}{l} \leftarrow \\ 1:4 \end{array} -5$$

$$\begin{pmatrix} 0 & 3 & 4 \\ -3 & 0 & -5 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow \underline{\underline{\text{Rank } A = 2}}$$

vaakevektorit $(0 \ 3 \ 4)$ ja $(3 \ 0 \ -5)$ ja pystyvektorit $(0 \ 3 \ 4)^T$ ja $(3 \ 0 \ -5)^T$ virittävät avaruuden.

L5

VKO 49 / C3-II / SN

$$A = \begin{pmatrix} 0 & 3 & -1 \\ -3 & 0 & 4 \\ 1 & 4 & 0 \end{pmatrix}$$

$$\det(A) = 0 \begin{vmatrix} 0 & 4 \\ 4 & 0 \end{vmatrix} - 3 \begin{vmatrix} -3 & 4 \\ 1 & 0 \end{vmatrix} + (-1) \begin{vmatrix} -3 & 0 \\ 1 & 4 \end{vmatrix}$$

$$= 0 - 3(0 - 4) + (-1)(-12 - 0) = 12 + 12 = \underline{\underline{24}}$$

"EXTRA":

$$A = \begin{pmatrix} 0 & 3 & -1 \\ -3 & 0 & -4 \\ 1 & 4 & 0 \end{pmatrix}$$

$$\det(A) = 0 \begin{vmatrix} 0 & -4 \\ 4 & 0 \end{vmatrix} - 3 \begin{vmatrix} -3 & -4 \\ 1 & 0 \end{vmatrix} - 1 \begin{vmatrix} -3 & 0 \\ 1 & 4 \end{vmatrix}$$

$$= 0 - 3(0 - (-4 \cdot 1)) - 1(-12) = -12 + 12 = \underline{\underline{0}}$$

L6

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 4 & 1 \end{pmatrix}$$

Vko 44/C3-II/S1

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 5 & 4 & 1 & 0 & 0 & 1 \end{array} \right) \begin{array}{l} \leftarrow -2 \\ \leftarrow -5 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 4 & 1 & -5 & 0 & 1 \end{array} \right) \leftarrow -4$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & 3 & -4 & 1 \end{array} \right)$$

$$= A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & -4 & 1 \end{pmatrix}$$

$$A A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 2-2 & 1 & 0 \\ 5-8+3 & 4-4 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

□