HARMONIC ANALYSIS, 8. EXERCISE

1. (3p) Prove Lemma 6.18 in the lecture note, i.e. that the maximal operator $H^{(*)}$ is weak $(1,1)$ in $S(\mathbf{R})$, by verifying Steps 1-3 stated after the statement of the lemma. Every step is independent and worth of 1 point. One can proceed for example to Step 2 without showing Step 1.
2. (2p) Prove Theorem 6.19 in the lecture note, i.e. that if $f \in L^1(\mathbf{R})$, then the limit
$$(Hf)(x) = \lim_{\varepsilon \downarrow 0} (H^{(\varepsilon)} f)(x)$$
exists for almost every $x \in \mathbf{R}$ and there is a constant c such that
$$|\{x \in \mathbf{R} \ : \ |(Hf)(x)| > \lambda\}| \leq \frac{c_p}{\lambda} \|f\|_1$$
for all $\lambda > 0$.

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