HARMONIC ANALYSIS, 11. EXERCISE

In these exercises, T stands for the singular integral operator of convolution type satisfying properties $(i - iv)$.

1. (3p.) Finish the proof of Theorem 6.35 in the lecture note following the lines described in Step 3 in the proof.
2. (1p.) Show that there is a constant $c = c(n, A_1, A_2)$ such that
$$\int_{\mathbf{R}^n \cap \{|y|>2|x|\}} \left| \frac{\Omega(y-x)}{|y-x|^n} - \frac{\Omega(y)}{|y|^n} \right| dy \leq c$$
for all $x \in \mathbf{R}^n$.
3. (1p.) Let $\varepsilon > 0$ and suppose that $f \in S(\mathbf{R}^n)$. Let $x_0 \in \mathbf{R}^n$, $\varrho > 0$, and $g = (1 - \chi_{B(x_0,25\varrho)})f$. Show that there is a constant $c = c(n, A_1, A_2)$ such that
$$\left|T^{(\varepsilon)}g(z) - T^{(\varepsilon)}g(x)\right| \leq cMf(x), \qquad |Tg(z) - Tg(x)| \leq cMf(x)$$
for all $x, z \in B(x, 5\varrho)$.

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