

■ Alternative way:

```
Needs["VectorAnalysis`"]

GenEx1 = {AA[Xx, Yy, Zz], BB[Xx, Yy, Zz], CC[Xx, Yy, Zz]};
GenEx2 = {DD[Xx, Yy, Zz], EE[Xx, Yy, Zz], FF[Xx, Yy, Zz]};
h0 = GenEx1 . Curl[GenEx2]

AA[Xx, Yy, Zz] (-EE(0,0,1)[Xx, Yy, Zz] + FF(0,1,0)[Xx, Yy, Zz]) +
CC[Xx, Yy, Zz] (-DD(0,1,0)[Xx, Yy, Zz] + EE(1,0,0)[Xx, Yy, Zz]) +
BB[Xx, Yy, Zz] (DD(0,0,1)[Xx, Yy, Zz] - FF(1,0,0)[Xx, Yy, Zz])
```

■ From scratch

```
coords = {Xx, Yy, Zz};
extDer[form_, i_, j_] := D[form[[j]], coords[[i]]] - D[form[[i]], coords[[j]]]
hel[formA_, formB_] := Module[{},
  res = 1/2 Sum[
    formA[[ii]] extDer[formB, jj, kk] Signature[{ii, jj, kk}]
    ,
    {ii, 1, 3},
    {jj, 1, 3},
    {kk, 1, 3}];
  Simplify[res]
]
Simplify[hel[GenEx1, GenEx2] - h0]
0
```

■ First example of compatible forms

```
alpha = {0, ConstC Xx, 1}; (* alpha = dz + C x dy *)
beta = {ConstD Yy, 0, 1} (* beta = dz + D y dx *)
{ConstD Yy, 0, 1}

(* alpha v dalpha *)
hel[alpha, alpha]
ConstC

(* alpha v dbeta *)
hel[alpha, beta]
-ConstD

(* beta v dalpha *)
hel[beta, alpha]
ConstC

(* beta v dbeta *)
hel[beta, beta]
-ConstD
```

■ Example of contact forms that are not compatible

```
beta = {0, lambda1 Sin[K Xx], Cos[K Xx]}; (* lambda = +1/-1 *)
gamma = {0, -Cos[K Xx], lambda1 Sin[K Xx]}; (* lambda = +1/-1 *)
```

```

(* alpha v dalpha *)
hel[beta, beta]
K lambda1

(* alpha v dbeta *)
hel[beta, gamma]
% /. {lambda1^2 -> 1}

1
- - K (-1 + lambda1^2) Sin[2 K Xx]
2

0

(* beta v dalpha *)
hel[gamma, beta]
% /. {lambda1^2 -> 1}

1
- K (-1 + lambda1^2) Sin[2 K Xx]
2

0

(* beta v dbeta *)
hel[gamma, gamma]
K lambda1

```

■ Example of compatible contact forms that are compatible

```

beta = {0, pm Sin[kval Xx], Cos[kval Xx]};
gamma = {0, pm Sin[kval Xx + degα], Cos[kval Xx + degα]};

hel[beta, beta]
kval pm

hel[beta, gamma]
kval pm Cos[degα]

hel[gamma, beta]
kval pm Cos[degα]

hel[gamma, gamma]
kval pm

Simplify[ Sin[kx] Sin[kx + φ] + Cos[kx] Cos[kx + φ] ]
Cos[φ]

```