- **Check that conjugation**

  G -> Sigma.G.Sigma^T

  for area metrics corresponds to conjugation

  kappa -> Hodge(++++) o kappa o Hodge(++++)

  for (2,2)-tensors

In[1]:=
```
SetDirectory["/www/user/fdahl/papers/Conjugation/"];
<< kappaLib.m
<< Petrov.m
```

```
KappaLib v1.1

Petrov routine loaded
```

- **We only need the result for skewon-free (2,2)-tensors. However, we will prove the result for all (2,2)-tensors.**

In[4]:=
```
kappa = emGeneralKappa["k"];
```

In[5]:=
```
kappaMat = emKappaToMatrix[kappa];
kappaMat // MatrixForm
```

Out[6]//MatrixForm=

$$\begin{pmatrix} k11 & k12 & k13 & k14 & k15 & k16 \\ k21 & k22 & k23 & k24 & k25 & k26 \\ k31 & k32 & k33 & k34 & k35 & k36 \\ k41 & k42 & k43 & k44 & k45 & k46 \\ k51 & k52 & k53 & k54 & k55 & k56 \\ k61 & k62 & k63 & k64 & k65 & k66 \end{pmatrix}$$

- **Compute Petrov matrix for**

  Hodge(++++) o kappa o Hodge(++++)

In[7]:=
```
metric = DiagonalMatrix[{1, 1, 1, 1}];
hodge = emHodge[metric];

hkh = emCompose[hodge, emCompose[kappa, hodge]];
```

In[10]:=
```
Petrov[emKappaToMatrix[hkh]]
```

Out[10]//MatrixForm=

$$\begin{pmatrix} k41 & k42 & k43 & k46 & k45 & k44 \\ k51 & k52 & k53 & k56 & k55 & k54 \\ k61 & k62 & k63 & k66 & k65 & k64 \\ k31 & k32 & k33 & k36 & k35 & k34 \\ k21 & k22 & k23 & k26 & k25 & k24 \\ k11 & k12 & k13 & k16 & k15 & k14 \end{pmatrix}$$

In[11]:=
```
PetrovConjI = %;
```

▪ Compute conjugation of Petrov matrix by "Sigma" symbol.

In[12]:=
```
Petrov[kappaMat]
```

Out[12]//MatrixForm=

$$\begin{pmatrix} k14 & k15 & k16 & k13 & k12 & k11 \\ k24 & k25 & k26 & k23 & k22 & k21 \\ k34 & k35 & k36 & k33 & k32 & k31 \\ k64 & k65 & k66 & k63 & k62 & k61 \\ k54 & k55 & k56 & k53 & k52 & k51 \\ k44 & k45 & k46 & k43 & k42 & k41 \end{pmatrix}$$

In[13]:=
```
petrovMat = %;
petrovMat // MatrixForm
```

Out[14]//MatrixForm=

$$\begin{pmatrix} k14 & k15 & k16 & k13 & k12 & k11 \\ k24 & k25 & k26 & k23 & k22 & k21 \\ k34 & k35 & k36 & k33 & k32 & k31 \\ k64 & k65 & k66 & k63 & k62 & k61 \\ k54 & k55 & k56 & k53 & k52 & k51 \\ k44 & k45 & k46 & k43 & k42 & k41 \end{pmatrix}$$

In[15]:=
```
Sigma = Table[
   Signature[{aOrd[i][[1]], aOrd[i][[2]], aOrd[j][[1]], aOrd[j][[2]]}], {i, 1, 6}, {j, 1, 6}];
Sigma // MatrixForm
```

Out[16]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[17]:=
```
PetrovConjII = Sigma.petrovMat.Transpose[Sigma];
```

▪ Check that operations commute

In[18]:=
```
PetrovConjI - PetrovConjII
```

Out[18]=
```
{{0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}}
```

In[19]:=
```
NotebookPrint[SelectedNotebook[],
 "/www/user/fdahl/papers/Conjugation/notebooks/Conjugation.pdf"]
```