In[1]:=
```
SetDirectory["/www/user/fdahl/papers/Conjugation/"];
<< kappaLib.m
<< Petrov.m
```

```
KappaLib v1.1

Petrov routine loaded
```

■ Class XXI: (321)

In[4]:=
$$B = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix};$$

In[5]:=
$$V = \begin{pmatrix} lam1 & 1 & 0 & 0 & 0 & 0 \\ 0 & lam1 & 1 & 0 & 0 & 0 \\ 0 & 0 & lam1 & 0 & 0 & 0 \\ 0 & 0 & 0 & lam2 & 1 & 0 \\ 0 & 0 & 0 & 0 & lam2 & 0 \\ 0 & 0 & 0 & 0 & 0 & lam3 \end{pmatrix};$$

In[6]:=
```
Eigenvalues[V]
```

Out[6]=
```
{lam1, lam1, lam1, lam2, lam2, lam3}
```

In[7]:=
$$W = \begin{pmatrix} 0 & 0 & eps1 & 0 & 0 & 0 \\ 0 & eps1 & 0 & 0 & 0 & 0 \\ eps1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & eps2 & 0 \\ 0 & 0 & 0 & eps2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & eps3 \end{pmatrix};$$

■ Possible choices for eps_i:

In[8]:=
```
Eigenvalues[W]
```

Out[8]=
```
{-eps1, eps1, eps1, -eps2, eps2, eps3}
```

In[9]:=
```
s[i_] := (-1)^i
```

In[10]:=
```
For[i1 = 0, i1 ≤ 1, i1++,
 For[i2 = 0, i2 ≤ 1, i2++,
  For[i3 = 0, i3 ≤ 1, i3++,
   Print[ToString[{s[i1], s[i2], s[i3]}]];
   Print[Sort[Eigenvalues[W] /. {eps1 → s[i1], eps2 → s[i2], eps3 → s[i3]}]];
  ]
 ]
]
```

{1, 1, 1}

{-1, -1, 1, 1, 1, 1}

{1, 1, -1}

{-1, -1, -1, 1, 1, 1}

{1, -1, 1}

{-1, -1, 1, 1, 1, 1}

{1, -1, -1}

{-1, -1, -1, 1, 1, 1}

{-1, 1, 1}

{-1, -1, -1, 1, 1, 1}

{-1, 1, -1}

{-1, -1, -1, -1, 1, 1}

{-1, -1, 1}

{-1, -1, -1, 1, 1, 1}

{-1, -1, -1}

{-1, -1, -1, -1, 1, 1}

■ 4 sign possibilities: (e1,e2,e3)= {++-, +--, -++, --+}

In[11]:=
```
W = W /. {eps3 → -eps1};
W // MatrixForm
Eigenvalues[W]
```

Out[12]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & \text{eps1} & 0 & 0 & 0 \\ 0 & \text{eps1} & 0 & 0 & 0 & 0 \\ \text{eps1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \text{eps2} & 0 \\ 0 & 0 & 0 & \text{eps2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\text{eps1} \end{pmatrix}$$

Out[13]= {-eps1, -eps1, eps1, eps1, -eps2, eps2}

In[14]:=
```
(* Found using FindSPermutations.m *)
S = {{0, 0, 0, eps2, 0, 0}, {1, 0, 0, 0, 0, 0}, {0, 1/Sqrt[2], 0, 0, 0, -1/Sqrt[2]}, {0, 0, 0, 0, 1, 0}, {0, 0, eps1, 0, 0, 0}, {0, eps1/Sqrt[2], 0, 0, 0, eps1/Sqrt[2]}};
```

- **Note: S does not depend on eps3 since we always have eps3 = -eps1.**
- **Check that S is in the set mathcal(S)**

In[15]:=
```
Transpose[S].B.S == W
```

Out[15]= True

- **Compute result**

In[16]:=
```
res = Simplify[S.V.Inverse[S]];
res // MatrixForm
```

Out[17]//MatrixForm=

$$\begin{pmatrix} lam2 & 0 & 0 & eps2 & 0 & 0 \\ 0 & lam1 & \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}\ eps1} \\ 0 & 0 & \frac{lam1+lam3}{2} & 0 & \frac{1}{\sqrt{2}\ eps1} & \frac{lam1-lam3}{2\ eps1} \\ 0 & 0 & 0 & lam2 & 0 & 0 \\ 0 & 0 & 0 & 0 & lam1 & 0 \\ 0 & 0 & \frac{1}{2} eps1\ (lam1 - lam3) & 0 & \frac{1}{\sqrt{2}} & \frac{lam1+lam3}{2} \end{pmatrix}$$

In[18]:=
```
Petrov[res]
```

Out[18]//MatrixForm=

$$\begin{pmatrix} eps2 & 0 & 0 & 0 & 0 & lam2 \\ 0 & 0 & \frac{1}{\sqrt{2}\ eps1} & \frac{1}{\sqrt{2}} & lam1 & 0 \\ 0 & \frac{1}{\sqrt{2}\ eps1} & \frac{lam1-lam3}{2\ eps1} & \frac{lam1+lam3}{2} & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{lam1+lam3}{2} & \frac{1}{2} eps1\ (lam1 - lam3) & 0 & 0 \\ 0 & lam1 & 0 & 0 & 0 & 0 \\ lam2 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- **Export notebook as .pdf**

In[19]:=
```
NotebookPrint[SelectedNotebook[],
 "/www/user/fdahl/papers/Conjugation/notebooks/ClassXXI.pdf"]
```