In[1]:=
```
SetDirectory["/www/user/fdahl/papers/Conjugation/"];
<< kappaLib.m
<< Petrov.m
```

```
KappaLib v1.1

Petrov routine loaded
```

In[4]:=
$$V = \begin{pmatrix} lam1 & 1 & 0 & 0 & 0 & 0 \\ 0 & lam1 & 0 & 0 & 0 & 0 \\ 0 & 0 & lam2 & 1 & 0 & 0 \\ 0 & 0 & 0 & lam2 & 0 & 0 \\ 0 & 0 & 0 & 0 & lam3 & 0 \\ 0 & 0 & 0 & 0 & 0 & lam4 \end{pmatrix};$$

In[5]:=
$$B = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix};$$

$$W = \begin{pmatrix} 0 & eps1 & 0 & 0 & 0 & 0 \\ eps1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & eps2 & 0 & 0 \\ 0 & 0 & eps2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & eps3 & 0 \\ 0 & 0 & 0 & 0 & 0 & eps4 \end{pmatrix} /. \{eps3 \to -1,\ eps4 \to 1\};$$

■ Solve S starting from expression in SWW

In[7]:=
```
(* First find matrix that gives matrix in the SWW paper *)
```

$$Petrov\left[\begin{pmatrix} lam1 & 0 & 0 & 0 & 0 & 0 \\ 0 & lam2 & 0 & 0 & eps2 & 0 \\ 0 & 0 & \frac{1}{2}(-lam3 + lam4) & 0 & 0 & \frac{lam3+lam4}{2} \\ eps1 & 0 & 0 & lam1 & 0 & 0 \\ 0 & 0 & 0 & 0 & lam2 & 0 \\ 0 & 0 & \frac{lam3+lam4}{2} & 0 & 0 & \frac{1}{2}(-lam3 + lam4) \end{pmatrix}\right]$$

Out[7]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & lam1 \\ 0 & eps2 & 0 & 0 & lam2 & 0 \\ 0 & 0 & \frac{lam3+lam4}{2} & \frac{1}{2}(-lam3 + lam4) & 0 & 0 \\ 0 & 0 & \frac{1}{2}(-lam3 + lam4) & \frac{lam3+lam4}{2} & 0 & 0 \\ 0 & lam2 & 0 & 0 & 0 & 0 \\ lam1 & 0 & 0 & 0 & 0 & eps1 \end{pmatrix}$$

In[8]:=
```
S = Table[ToExpression["s" <> ToString[i] <> ToString[j]], {i, 1, 6}, {j, 1, 6}];
j1 = Flatten[Transpose[S].B.S - W];
j2 = Flatten[{{xx1, 0, 0, 0, 0, 0}, {0, xx2, 0, 0, eps2, 0}, {0, 0, xx3, 0, 0, xx4}, {eps1, 0, 0, xx1, 0, 0}, {0, 0, 0, 0, xx2, 0}, {0, 0, xx4, 0, 0, xx3}}.S - S.V];
exp = Union[Join[j2, j1]] /. {eps1 → 1, eps2 → 1};
```

In[11]:=
```
Simplify[exp] // MatrixForm
```

Out[11]//MatrixForm=

$$\begin{pmatrix}
2\,(s11\,s41 + s21\,s51 + s31\,s61) \\
-1 + s12\,s41 + s11\,s42 + s22\,s51 + s21\,s52 + s32\,s61 + s31\,s62 \\
2\,(s12\,s42 + s22\,s52 + s32\,s62) \\
s13\,s41 + s11\,s43 + s23\,s51 + s21\,s53 + s33\,s61 + s31\,s63 \\
s13\,s42 + s12\,s43 + s23\,s52 + s22\,s53 + s33\,s62 + s32\,s63 \\
2\,(s13\,s43 + s23\,s53 + s33\,s63) \\
s14\,s41 + s11\,s44 + s24\,s51 + s21\,s54 + s34\,s61 + s31\,s64 \\
s14\,s42 + s12\,s44 + s24\,s52 + s22\,s54 + s34\,s62 + s32\,s64 \\
-1 + s14\,s43 + s13\,s44 + s24\,s53 + s23\,s54 + s34\,s63 + s33\,s64 \\
2\,(s14\,s44 + s24\,s54 + s34\,s64) \\
s15\,s41 + s11\,s45 + s25\,s51 + s21\,s55 + s35\,s61 + s31\,s65 \\
s15\,s42 + s12\,s45 + s25\,s52 + s22\,s55 + s35\,s62 + s32\,s65 \\
s15\,s43 + s13\,s45 + s25\,s53 + s23\,s55 + s35\,s63 + s33\,s65 \\
s15\,s44 + s14\,s45 + s25\,s54 + s24\,s55 + s35\,s64 + s34\,s65 \\
1 + 2\,s15\,s45 + 2\,s25\,s55 + 2\,s35\,s65 \\
s16\,s41 + s11\,s46 + s26\,s51 + s21\,s56 + s36\,s61 + s31\,s66 \\
s16\,s42 + s12\,s46 + s26\,s52 + s22\,s56 + s36\,s62 + s32\,s66 \\
s16\,s43 + s13\,s46 + s26\,s53 + s23\,s56 + s36\,s63 + s33\,s66 \\
s16\,s44 + s14\,s46 + s26\,s54 + s24\,s56 + s36\,s64 + s34\,s66 \\
s16\,s45 + s15\,s46 + s26\,s55 + s25\,s56 + s36\,s65 + s35\,s66 \\
-1 + 2\,s16\,s46 + 2\,s26\,s56 + 2\,s36\,s66 \\
s11\,(-lam1 + xx1) \\
-s11 - lam1\,s12 + s12\,xx1 \\
s13\,(-lam2 + xx1) \\
-s13 - lam2\,s14 + s14\,xx1 \\
s15\,(-lam3 + xx1) \\
s16\,(-lam4 + xx1) \\
s11 + s41\,(-lam1 + xx1) \\
s12 - s41 - lam1\,s42 + s42\,xx1 \\
s13 + s43\,(-lam2 + xx1) \\
s14 - s43 - lam2\,s44 + s44\,xx1 \\
s15 + s45\,(-lam3 + xx1) \\
s16 + s46\,(-lam4 + xx1)
\end{pmatrix}$$

$$\begin{pmatrix}
s10 + s40\,(-\text{lam4} + xx1) \\
-\text{lam1}\,s21 + s51 + s21\,xx2 \\
-s21 - \text{lam1}\,s22 + s52 + s22\,xx2 \\
-\text{lam2}\,s23 + s53 + s23\,xx2 \\
-s23 - \text{lam2}\,s24 + s54 + s24\,xx2 \\
-\text{lam3}\,s25 + s55 + s25\,xx2 \\
-\text{lam4}\,s26 + s56 + s26\,xx2 \\
s51\,(-\text{lam1} + xx2) \\
-s51 - \text{lam1}\,s52 + s52\,xx2 \\
s53\,(-\text{lam2} + xx2) \\
-s53 - \text{lam2}\,s54 + s54\,xx2 \\
s55\,(-\text{lam3} + xx2) \\
s56\,(-\text{lam4} + xx2) \\
-\text{lam1}\,s61 + s61\,xx3 + s31\,xx4 \\
-s61 - \text{lam1}\,s62 + s62\,xx3 + s32\,xx4 \\
-\text{lam2}\,s63 + s63\,xx3 + s33\,xx4 \\
-s63 - \text{lam2}\,s64 + s64\,xx3 + s34\,xx4 \\
-\text{lam3}\,s65 + s65\,xx3 + s35\,xx4 \\
-\text{lam4}\,s66 + s66\,xx3 + s36\,xx4 \\
-\text{lam1}\,s31 + s31\,xx3 + s61\,xx4 \\
-s31 - \text{lam1}\,s32 + s32\,xx3 + s62\,xx4 \\
-\text{lam2}\,s33 + s33\,xx3 + s63\,xx4 \\
-s33 - \text{lam2}\,s34 + s34\,xx3 + s64\,xx4 \\
-\text{lam3}\,s35 + s35\,xx3 + s65\,xx4 \\
-\text{lam4}\,s36 + s36\,xx3 + s66\,xx4
\end{pmatrix}$$

In[12]:=
```
sub = {s11 → 0, s13 → 0, s15 → 0, s16 → 0, s51 → 0, s53 → 0, s55 → 0, s56 → 0, s12 → s41, s14 → s43,
    s52 → s21, s54 → s23, s61 → 0, s63 → 0, s31 → 0, s33 → 0, (* revert here *)s21 → 1,
    s41 → 0 , xx2 → lam1, s23 → 0, s43 → 1, xx1 → lam2, s24 → 0, s25 → 0, s26 → 0, s45 → 0,
    s46 → 0, s42 → 0, (* rev *) s22 → 0, (* compute Groebner basis below *)
    xx3 → (lam3 + lam4) / 2, s32 → 0, s34 → 0, s62 → 0, s64 → 0, s44 → 0, s66 → 1 / Sqrt[2],
    s36 → 1 / Sqrt[2], s35 → -s65, s65 → 1 / Sqrt[2], xx4 → (lam4 - lam3) / 2};
Simplify[Sort[Union[exp //. sub], emSize[#1] ≤ emSize[#2] &]] // MatrixForm
S //. sub // MatrixForm
```

Out[13]//MatrixForm=

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Out[14]//MatrixForm=

$$\begin{pmatrix}
0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{pmatrix}$$

In[15]:=
```
(* see above *)
GroebnerBasis[Simplify[Sort[Union[exp //. sub], emSize[#1] ≤ emSize[#2] &]], Variables[S]]
```

Out[15]= { }

■ Put in eps_i

In[16]:=
$$Sx = \begin{pmatrix} 0 & 0 & 0 & a1 & 0 & 0 \\ a2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{a7}{\sqrt{2}} & \frac{a8}{\sqrt{2}} \\ 0 & 0 & a3 & 0 & 0 & 0 \\ 0 & a4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{a5}{\sqrt{2}} & \frac{a6}{\sqrt{2}} \end{pmatrix} /.$$

```
    {a5 → 1, a6 → 1, a7 → 1, a8 → 1, a1 → 1, a2 → 1, a3 → eps2, a4 → eps1};
Union[Flatten[Transpose[Sx].B.Sx - W]]
```

Out[17]= {0}

In[18]:=
```
Sx // MatrixForm
```

Out[18]//MatrixForm=
$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & eps2 & 0 & 0 & 0 \\ 0 & eps1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

In[19]:=
```
(* export .pdf *)
NotebookPrint[SelectedNotebook[],
 "/www/user/fdahl/papers/Conjugation/notebooks/ClassXV_Solve.pdf"]
```