# STOCHASTIC ANALYSIS: AN INTRODUCTION

ESKO VALKEILA

Abstract. The purpose of these lectures is to introduce stochastic integrals with respect to standard Brownian motion, or more generally with respect to continuous square integrable martingales. Before this we discuss discrete time parameter martingales to be familiar with some of the techniques needeed later. We also prove some results for discrete time parameter martingales, like moment inequalities and the important martingale convergence theorem. After this, and after defining stochastic integrals, we give some classical applications of the fundamental Itô formula: Lévy theorem to characterize Brownian motion and Girsanov theorem. Another application are the iterated integrals with respect to Brownian motion; these are useful for example in Malliavin calculus. We try to cover also the basic facts about stochastic differential equations. If the time permits, we discuss some other applications.

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