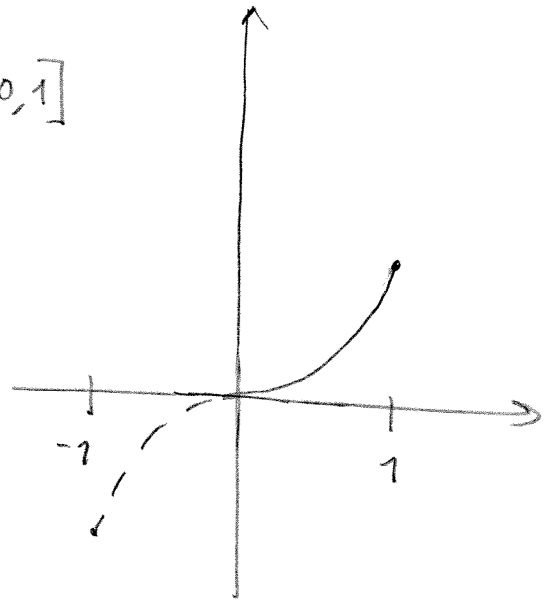
K3/P3 syksy 2005 11LV-mallinratkaisut

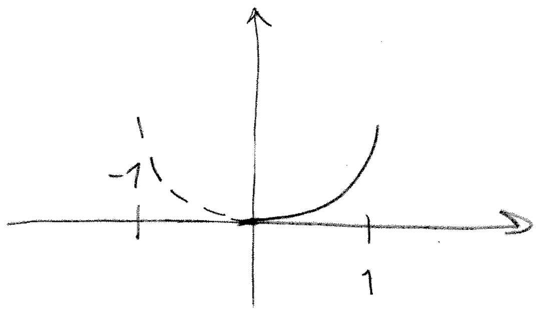
11.L1 (a) $f(x) = x^2, \quad x \in [0,1]$



Pariton jatke:

$$\tilde{f}(x) = \begin{cases} x^2, & x \in [0,1] \\ -f(-x) = \underline{\underline{-x^2}} & x \in [-1,0] \end{cases}$$

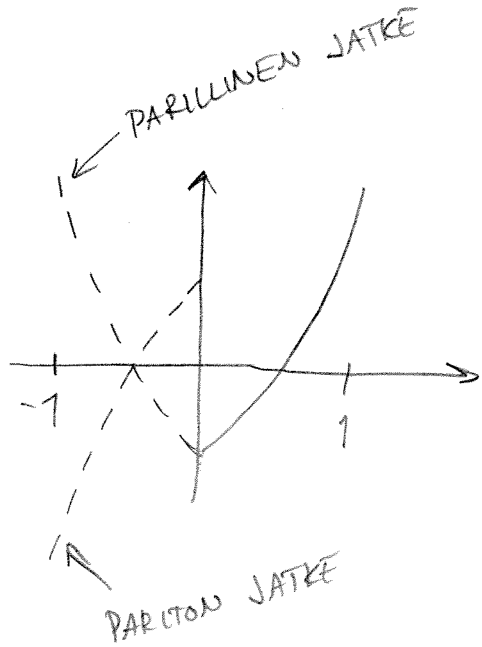
(b) $f(x) = x^3, \quad x \in [0,1]$



Parillinen jatke:

$$\hat{f}(x) = \begin{cases} x^3, & x \in [0,1] \\ f(-x) = \underline{\underline{-x^3}}, & x \in [-1,0] \end{cases}$$

(c) $f(x) = xe^x - e^{-x}, \quad x \in [0,1]$



Parillinen jatke:

$$\hat{f}(x) = \begin{cases} xe^x - e^{-x}, & x \in [0,1] \\ f(x) = \underline{\underline{-xe^{-x} - e^x}}, & x \in [-1,1] \end{cases}$$

Pariton jatke:

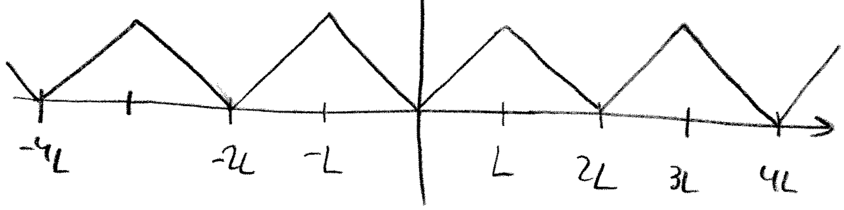
$$\tilde{f}(x) = \begin{cases} xe^x - e^{-x}, & x \in [0,1] \\ -f(-x) = \underline{\underline{xe^{-x} + e^x}}, & x \in [-1,1] \end{cases}$$

11.L2 $f(x) = x, \quad 0 < x < L$

(a) PARILLINEN JATKE

$$\hat{f}(x) = \begin{cases} x, & 0 < x < L \\ -x, & -L < x < 0 \end{cases}$$

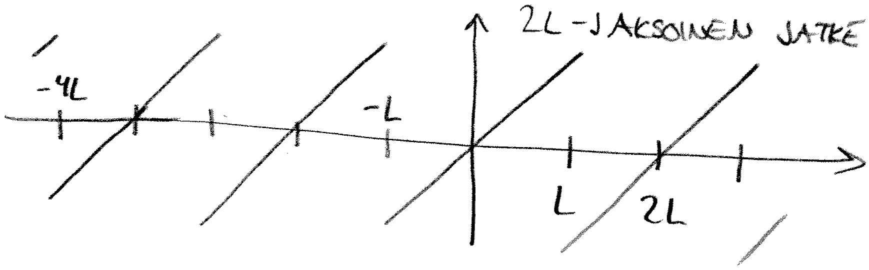
PARILLISEN JATKEEN 2L-JAKSOINEN JATKE



(b) PARITON JATKE

$$\tilde{f}(x) = x, \quad -L < x < L$$

PARITTOMAN JATKEEN 2L-JAKSOINEN JATKE



Fourier-sarja: $\tilde{f}(x) \sim a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$

$$a_0 = \frac{1}{2L}\int_{-L}^{L} x\,dx = 0$$

$$a_n = \frac{1}{L}\int_{-L}^{L} x\cos\frac{n\pi x}{L}\,dx = \underbrace{\frac{1}{L}\Big/_{-L}^{L} x\sin\frac{n\pi x}{L}\cdot\frac{L}{n\pi}}_{= L\sin n\pi - (-L\underbrace{\sin -n\pi}_{=-\sin n\pi}) = 0} - \frac{1}{L}\int_{-L}^{L}\sin\frac{n\pi x}{L}\,dx\,\frac{L}{n\pi}$$

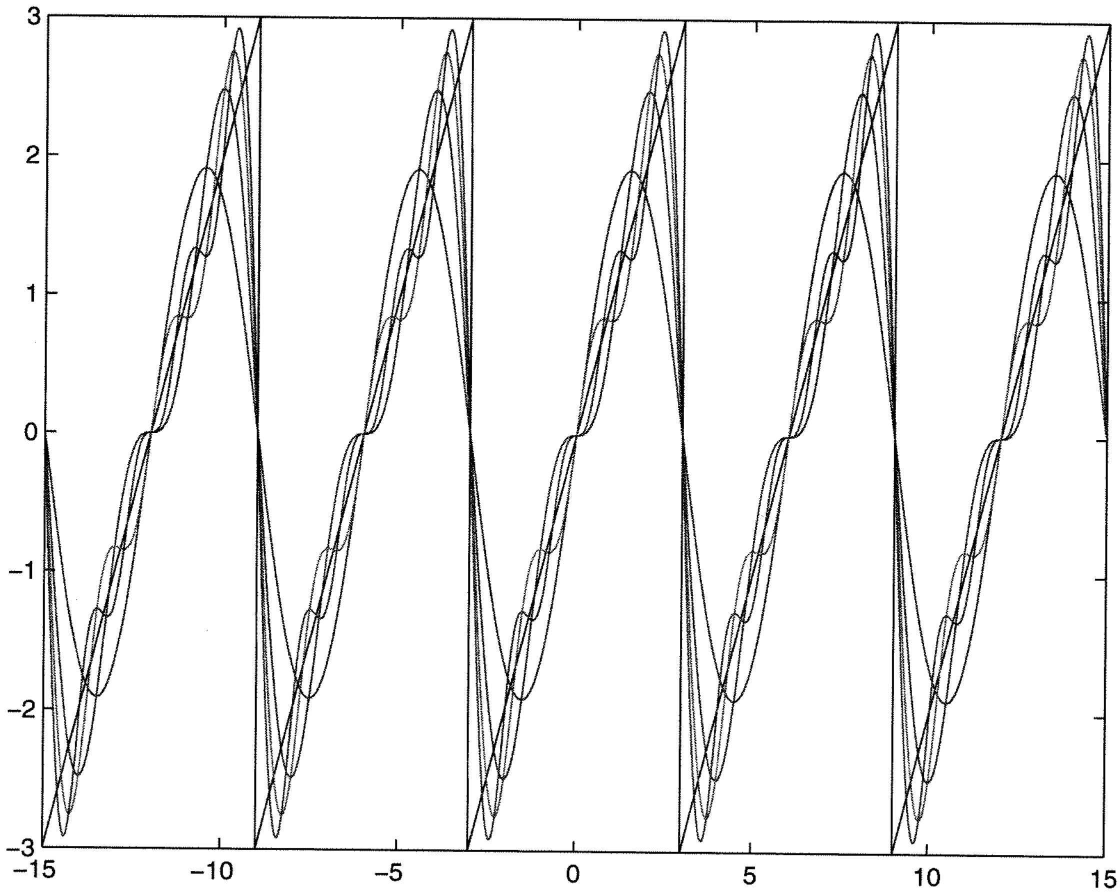
$$= -\frac{L}{n^2\pi^2}\Big/_{-L}^{L}\cos\frac{n\pi x}{L} = 0$$

$$b_n = \frac{1}{L}\int_{-L}^{L} x\sin\frac{n\pi x}{L}\,dx = \frac{1}{L}\Big/_{-L}^{L} x\cos\frac{n\pi x}{L}\cdot\left(-\frac{L}{n\pi}\right) + \frac{1}{n\pi}\int_{-L}^{L}\cos\frac{n\pi x}{L}\,dx\,\frac{L}{n\pi}$$

$$= -\frac{1}{n\pi}\Big(L\underbrace{\cos n\pi}_{=(-1)^n} - (-L)\underbrace{\cos -n\pi}_{=\cos n\pi = (-1)^n}\Big) + \underbrace{\frac{L}{n^2\pi^2}\Big/_{-L}^{L}\sin\frac{n\pi x}{L}}_{=0} = \frac{(-1)^{n-1}2L}{n\pi}$$

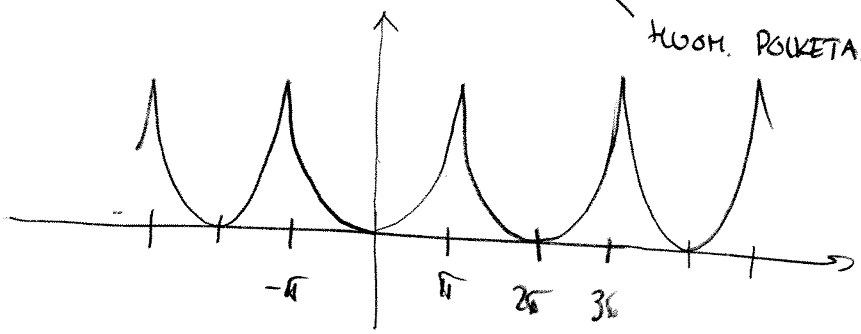
$$\underline{\underline{f(x) \sim \frac{2L}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}\sin\frac{n\pi x}{L}}} = \frac{2L}{\pi}\left[\sin\frac{n\pi x}{L} - \frac{1}{2}\sin\frac{2\pi x}{L} + \frac{1}{3}\sin\frac{3\pi x}{L} - \ldots\right]$$

```
L = 3;
x0 = [-L : .01 : L - 0.01];
f = [x0 x0 x0 x0 x0];
x = [-5*L : .01 : 5*L - .01];
f1 = 2*L/pi*sin(pi*x/L);
f2 = f1 - 2*L/pi*sin(2*pi*x/L)/2;
f3 = f2 + 2*L/pi*sin(3*pi*x/L)/3;
f4 = f3 - 2*L/pi*sin(4*pi*x/L)/4;
plot(x, f, x, f1, x, f2, x, f3, x, f4)
```



11.L3 Funktion $f(x) = \frac{x^2}{4}$ $(x \in [-\pi, \pi])$ 2π-jaksollinen jatke:

HUOM. POIKETAAN VÄHÄN TEHTÄVÄNANNOSTA



Jaksollinen jatke on jatkuva päätepisteissä, joten jaksollinen jatke $g(x)$ $(x \in \mathbb{R})$ on jatkuva kaikkialla (erityisesti siis paloittain jatkuva).
Lisäksi g:llä on kaikkialla vasemman- ja oikeanpuoleiset derivaatat.

$\Rightarrow$ Fourier-sarja $h(x) = \frac{\pi^2}{12} - \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \cos nx$ suppenee kaikissa pisteissä kohti arvoa $\frac{g(x-)+g(x+)}{2} = g(x)$.

KOSKA g ON JATKUVA

$$h(0) = \frac{\pi^2}{12} - \sum_{n=1}^{\infty} \frac{\overbrace{(-1)^{n+1}}^{=(-1)^{n-1}}}{n^2} \underbrace{\cos 0}_{=1} = g(0) = f(0) = 0$$

$$\Rightarrow \frac{\pi^2}{12} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$$

$$h(\pi) = \frac{\pi^2}{12} - \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \underbrace{\cos n\pi}_{=(-1)^n} = g(\pi) = f(\pi) = \frac{\pi^2}{4} \Rightarrow \frac{\pi^2}{6} = \frac{\pi^2}{4} - \frac{\pi^2}{12} = \sum_{n=1}^{\infty} \frac{1}{n^2}$$