Mat-5.3741 Theory of Elasticity (5 cp) L **Spring 2007**

Stenberg/Juntunen

Exercise 7

Problem 1

(Diaz-Weinsteins formula) In the de Saint-Venant torsion theory the torsionla rigidity is

$$J = \int_\Omega \left[\left(\frac{\partial \psi}{\partial x_1} - x_2 \right)^2 + \left(\frac{\partial \psi}{\partial x_2} + x_1 \right)^2 \right] dA.$$

Show that this can also be written as

$$J = I_p - \int_\Omega |\nabla \psi|^2 dA,$$

in which

$$I_p = \int_\Omega \left(x_1^2 + x_2^2 \right) dA$$

is the polar moment.

Problem 2

Check the example 4 in the book (p. 168) .

Problem 3 **(home exercise++)**

Compute by Fourier-series the torsional rigidity for the rectangular cross section $[-a, a] \times [-b, b]$. Study the convergence of the series and the value of the torsional rigidity as a function of the $[n, m]$-partial sum.

For the case when $b = t/2$ is "very small", one usually uses the "Föppls fomula"

$$J = \frac{1}{3} s t^3,$$

where $s = 2a$. Study the accuray of this formula with the aid of your Fourier-series solution.