Mat-1.3379 Dynaamisten systeemien erikoiskurssi **Eirola/Hannukainen**
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Tehtävät 1 ja 2 ovat kotitehtäviä . Kotitehtävät palautetaan laskuharjoituksiin mennessä huoneen Y323b edessä sijaitsevaan lokeroon tai laskuharjoitusten alussa assistentille.

1. (Problem 4.3 p.101)(Kotitehtävä) Show that every vector $\boldsymbol{u} \in E_-$ is of the form $\boldsymbol{u} = \boldsymbol{w} - \boldsymbol{S}\boldsymbol{w}$.

2. (Problem 4.4 p. 101)(Kotitehtävä) Take the $\boldsymbol{u}_{\{i,j\}}$ –basis of E_- and number it according to $\{i,j\} = \frac{1}{2}(i-1)(i-2)+j$. Check that this gives numbers $1, 2, \ldots, \frac{1}{2}n(n-1)$. Show that the matrix of $\boldsymbol{A} \odot \boldsymbol{B}$ in this basis is given by
$$\left(\boldsymbol{A} \odot \boldsymbol{B} \right)_{\{i,j\},\{k,l\}} = \frac{1}{2} \left(\begin{vmatrix} a_{i,k} & a_{i,l} \\ b_{j,k} & b_{j,l} \end{vmatrix} + \begin{vmatrix} b_{i,k} & b_{i,l} \\ a_{j,k} & a_{j,l} \end{vmatrix} \right) .$$
Hint: $\left(\boldsymbol{A} \odot \boldsymbol{B} \right)_{\{i,j\},\{k,l\}} = \boldsymbol{u}_{\{i,j\}}^T \left(\boldsymbol{A} \odot \boldsymbol{B} \right) \boldsymbol{u}_{\{k,l\}}$

3. (Example 5.16 p. 110) Consider the system
$$\begin{cases} x' &= -\sigma x + \sigma y \\ y' &= \rho x - xz - y \\ z' &= xy - \beta z \end{cases} ,$$
and let ρ be the free parameter, while σ and β are fixed, positive, values (the "classical" choice is $\sigma = 16$ and $\beta = 4$). Consider the equilibria problem $\boldsymbol{f}(\boldsymbol{u}) = 0$, $\boldsymbol{u} = (x, y, z, \rho)$, where $\boldsymbol{f}$ is the right hand side function. Take the trivial equilibrium, and find a branch point for it. Show that this is a pitchfork bifurcation. Which symmetry(ies) in the problem make it possible for this bifurcation to persist under perturbations respecting the symmetry(ies)?

4. (Problem 5.2 p. 135) Consider the map $\boldsymbol{f} = \mathrm{id} - \boldsymbol{g}$, where $\boldsymbol{g} : \mathbb{R}^n \to \mathbb{R}^n$ and id is the identity map in $\mathbb{R}^n$. Show that if $\boldsymbol{g}$ is a contraction, then $\boldsymbol{f}$ is a homeomorphism.

5. Consider the system
$$\begin{bmatrix} x^+ \\ y^+ \end{bmatrix} = \boldsymbol{\phi}(x,y) = \begin{bmatrix} \alpha x(1-x) - xy \\ \frac{1}{\beta} xy \end{bmatrix} ,$$
which is a discrete time version of a standard predator-prey model. Prove, that a nontrivial fixed point of the map undergoes a Neimark-Sacker bifurcation on a curve in (α, β)-plane, and compute the direction of the closed invariant-curve bifurcation. Guess what happens to the emergent closed invariant curve for parameter values far from the bifurcation one.