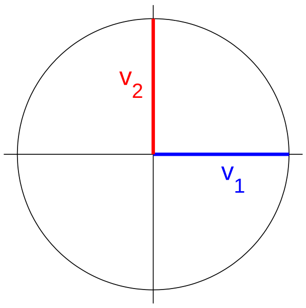
Matriisilaskennan sovellukset, syksy 2012 Harjoitus 5 (10.–14.12.2012)
Applications of matrix computations, fall 2012 Exercise 5 (10.–14.12.2012)

Samuli Siltanen

Note that this exercise has several pages!

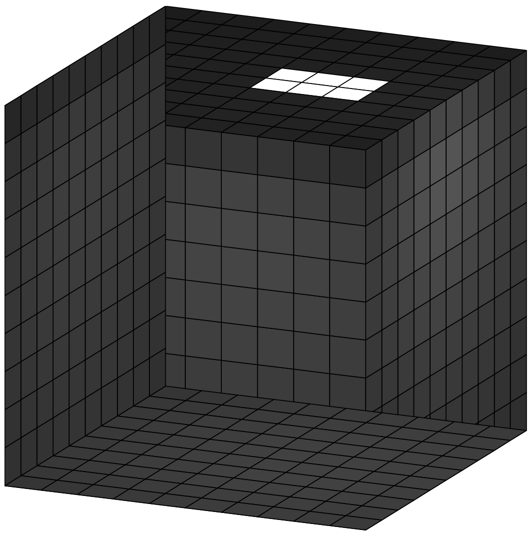
1. *Iterative solution using GMRES.* Consider the system of equations $Ax = b$, where
$$A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \qquad b = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}.$$
   (a) Solve x analytically by hand.
   (b) Solve x using Matlab and the command `inv`.
   (c) Solve x using Matlab and the GMRES method. Do this by modifying the files `Aaction.m` and `gmrestest.m`.

2. *Geometric illustration of eigenvectors.* Run the file `circle.m`. You should see something like this:



Define
$$B = \begin{bmatrix} 7/6 & -1/3 \\ 1/3 & 1/3 \end{bmatrix}.$$
   (a) Determine the eigenvalues and eigenvectors of B using the command `eig`.
   (b) Modify the file `circle.m` so that the picture shows the image of the unit disc under the transformation defined by B. Draw the two eigenvectors to the same image as red and blue lines.

3. Run the file `radiosity5.m`. You should see something like this:



    (a) Study the effect of parameter `sc_par` defined on line 56 in `radiosity5.m` by changing its value and looking at the results. Explain the changes you see in the image as a result of changing `sc_par`.
    (b) Same as (a), but with the parameter `cut_param` on line 269 in `radiosity5.m`.
    (c) Same as (a), but with the parameter `gammacorr` on line 276 in `radiosity5.m`.

4. Add another lamp to the room, on the left-hand side wall, and compute the lighting.

5. Modify the file `radiosity5.m` by using the QR decomposition in the solution on line 261.

6. Modify the file `radiosity5.m` by adding the effect of a front wall. Do not draw the front wall, but just include its effect in the construction of matrix F. Note that the new matrix F has size $6n^2 \times 6n^2$.

7. (This extra exercise requires a lot of work and is not required for passing the course.) Continue the previous exercise by adding a "table" to the room. The table is the square patch defined by $-\frac{1}{2} \leq x \leq \frac{1}{2}$ and $-\frac{1}{2} \leq y \leq \frac{1}{2}$ and $z = -\frac{1}{2}$. You will need to check "visibilities" by computing if a given vector connecting two centerpoints of pixels travels through the table. In such case the corresponding element of matrix F has to be zero. Compute the lighting and observe the soft shadow under the table.