Aalto-yliopisto
Teknillinen korkeakoulu

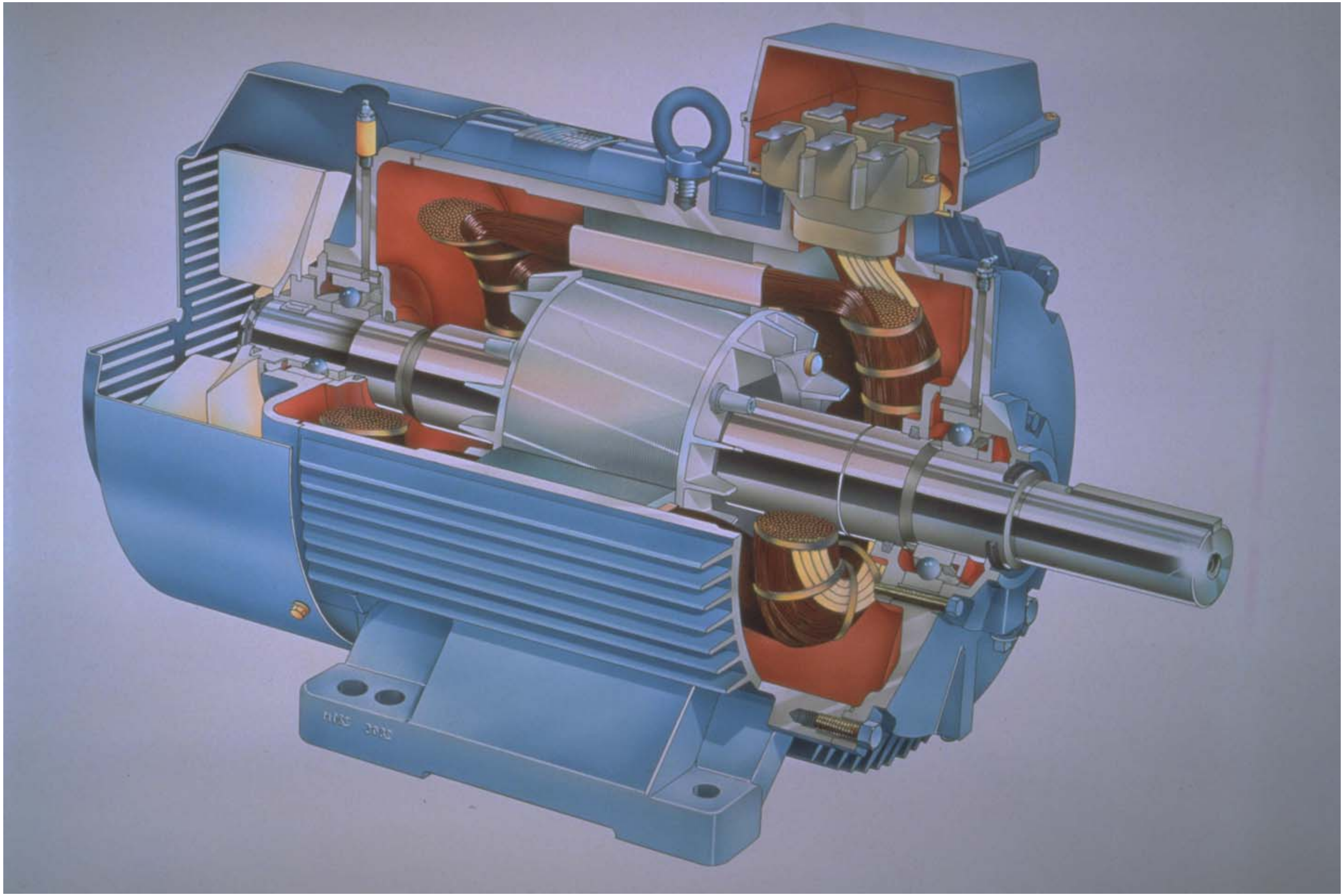
Mat-1.3656 Seminar on numerical analysis and computational science

Modelling of electrical machines

Antero Arkkio

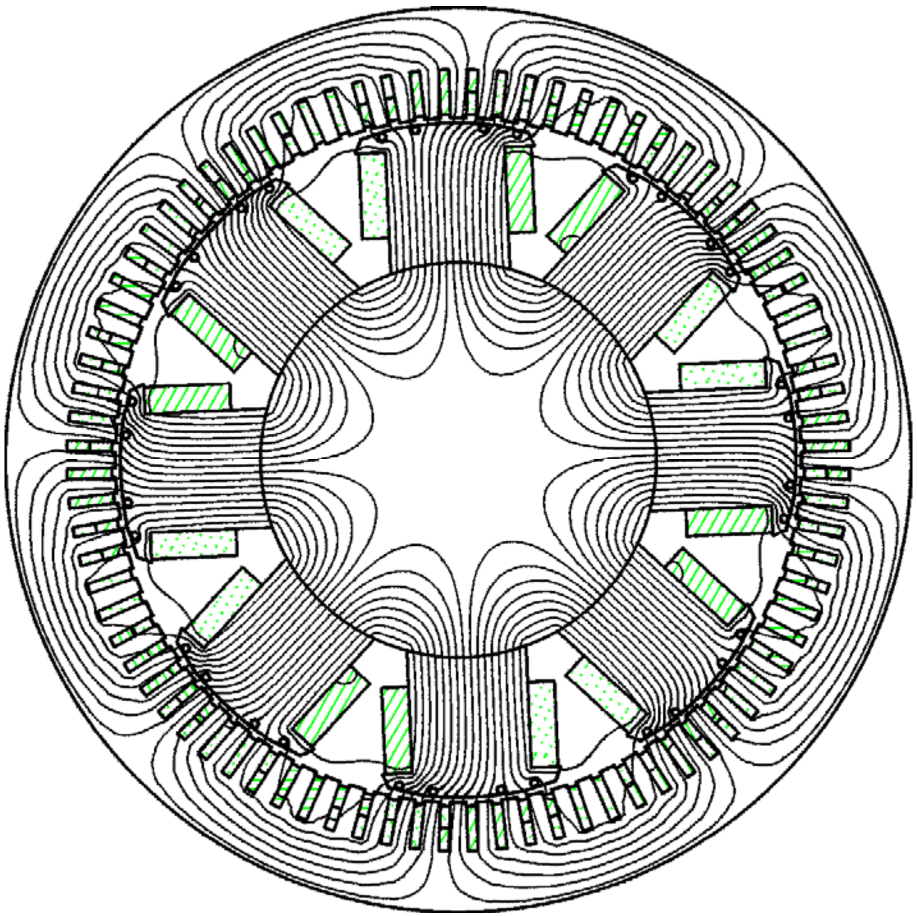
29/03/2010

Small cage induction motor

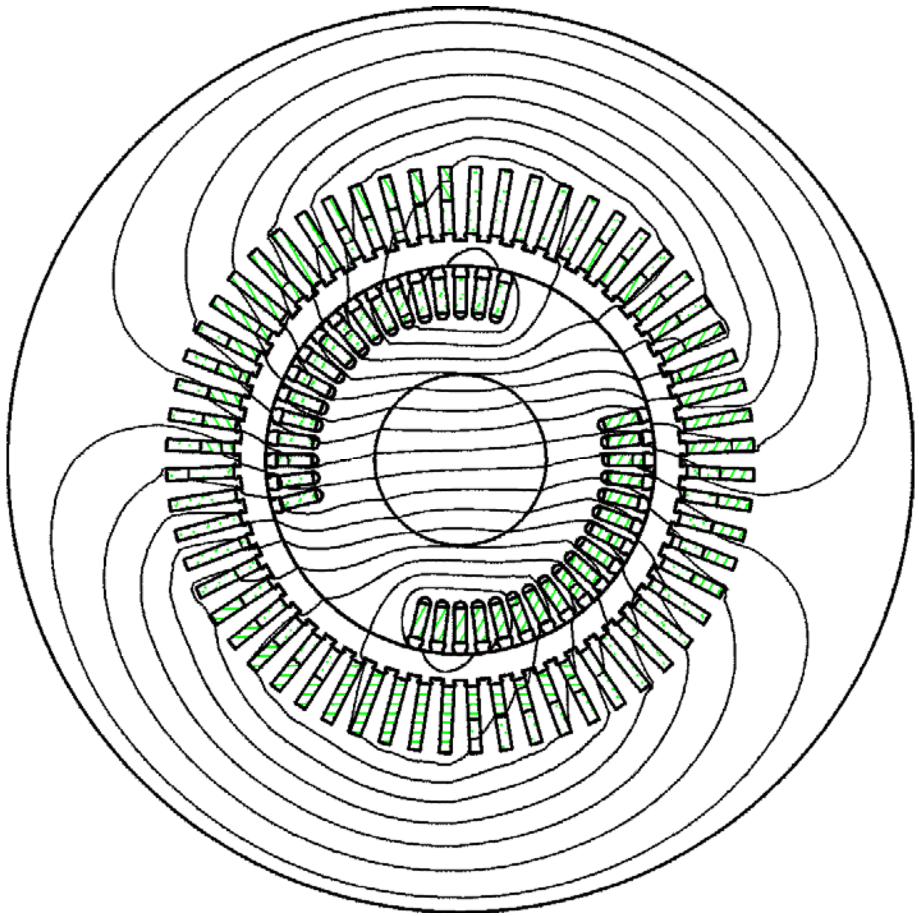


Synchronous machines

Over 99% of the electrical energy used in Finland is produced in rotating electrical machines.



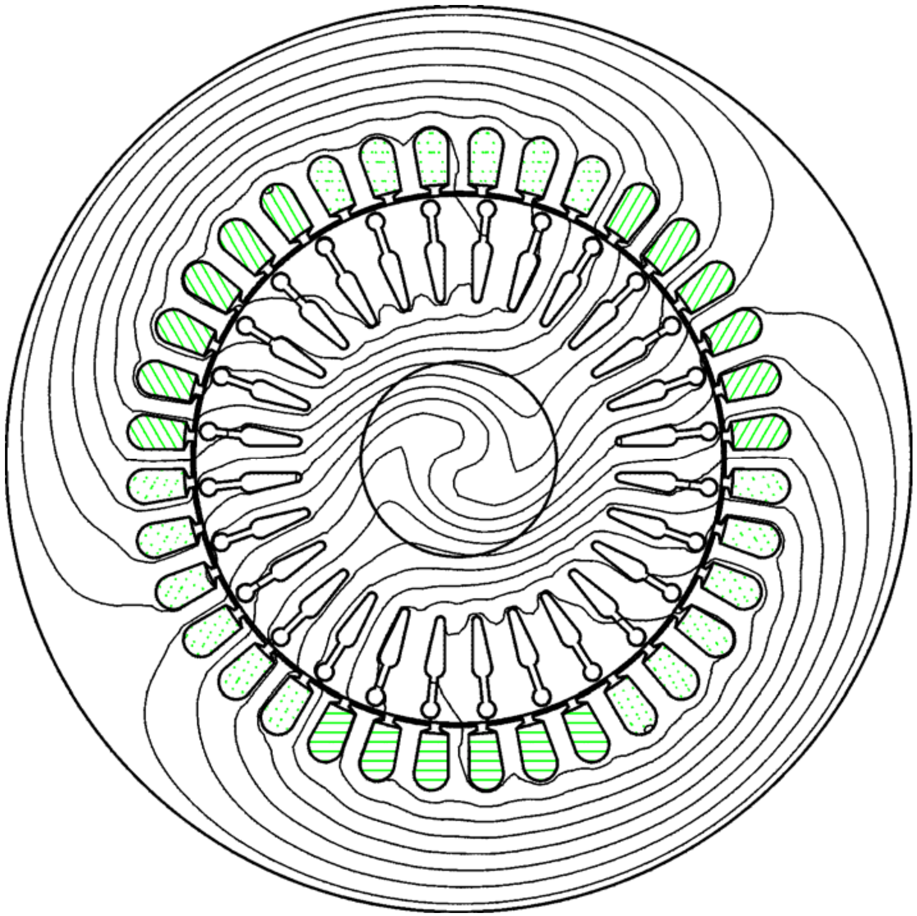
8 MW diesel-generator



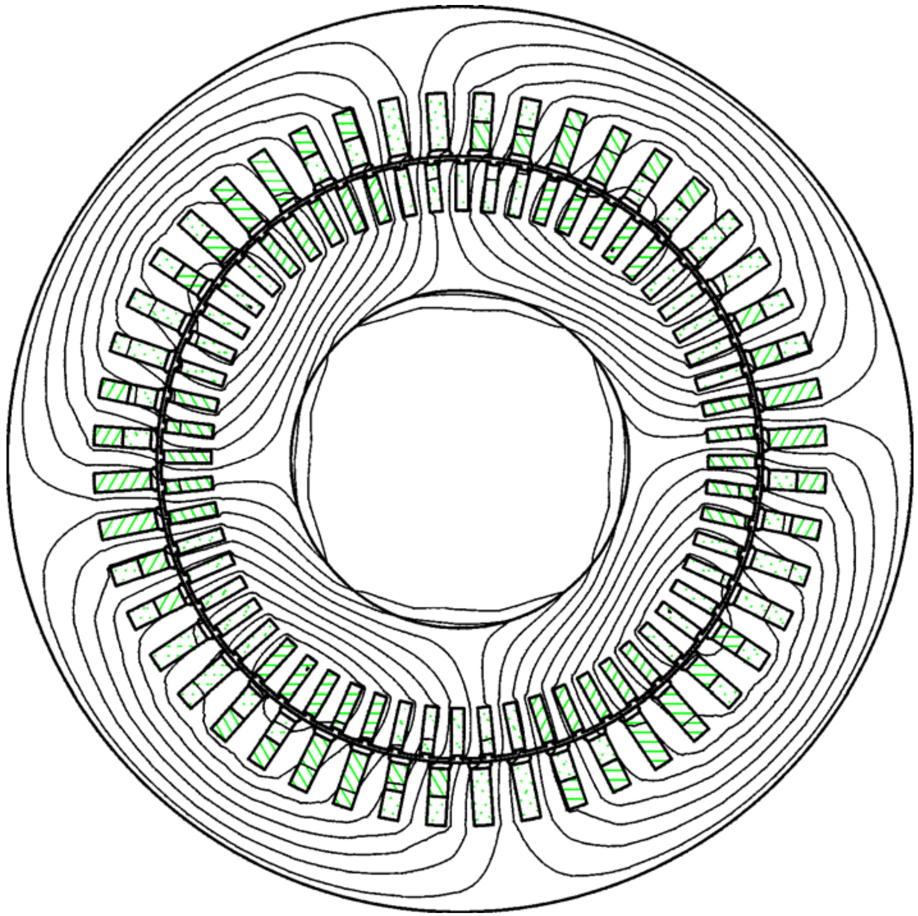
270 MVA turbo-generator

Induction machines

About 60% of the electrical energy is consumed by rotating electrical machines.



30 kW cage induction motor



1.7 MVA slip-ring generator

Basic equations of electromagnetic field

Five field variables (***E, D, H, B, J***) are needed to present a complete electromagnetic field.

Maxwell's equations

$$\nabla \cdot \boldsymbol{D} = \rho$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$$

Material equations

$$\boldsymbol{D} = \varepsilon \boldsymbol{E}$$

$$\boldsymbol{J} = \sigma \boldsymbol{E}$$

$$\boldsymbol{B} = \mu \boldsymbol{H}$$

In addition, the boundary conditions are needed.

$A\phi$-formulation

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} \qquad \nabla \cdot \boldsymbol{B} = 0 \qquad \nabla \times \boldsymbol{H} = \boldsymbol{J} \qquad \boldsymbol{H} = \nu \boldsymbol{B}$$

$$\boxed{\nabla \times \left(\nu \nabla \times \boldsymbol{A} \right) = \boldsymbol{J}}$$

Eddy-current problems: $\boldsymbol{J}$ is unknown.

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} = -\frac{\partial \left(\nabla \times \boldsymbol{A} \right)}{\partial t} = -\nabla \times \frac{\partial \boldsymbol{A}}{\partial t} \qquad \Rightarrow \qquad \boldsymbol{E} = -\frac{\partial \boldsymbol{A}}{\partial t} - \nabla \phi$$

as identically $\nabla \times \left(\nabla \phi \right) \equiv 0$. ϕ is the reduced scalar potential of electric field.

Current density and the **partial differential equation** of vector potential

$$\boldsymbol{J} = \sigma \boldsymbol{E} = -\sigma \frac{\partial \boldsymbol{A}}{\partial t} - \sigma \nabla \phi \qquad \boxed{\nabla \times \left(\nu \nabla \times \boldsymbol{A} \right) = -\sigma \frac{\partial \boldsymbol{A}}{\partial t} - \sigma \nabla \phi}$$

$A\phi$-formulation II

Eddy-current regions: $$\boldsymbol{J} = -\sigma \frac{\partial \boldsymbol{A}}{\partial t} - \sigma \nabla \phi$$

Source-current regions: $$\boldsymbol{J} = \boldsymbol{J}_s$$

Combined equation: $$\nabla \times (\nu \nabla \times \boldsymbol{A}) + \sigma \frac{\partial \boldsymbol{A}}{\partial t} + \sigma \nabla \phi = \boldsymbol{J}_s$$

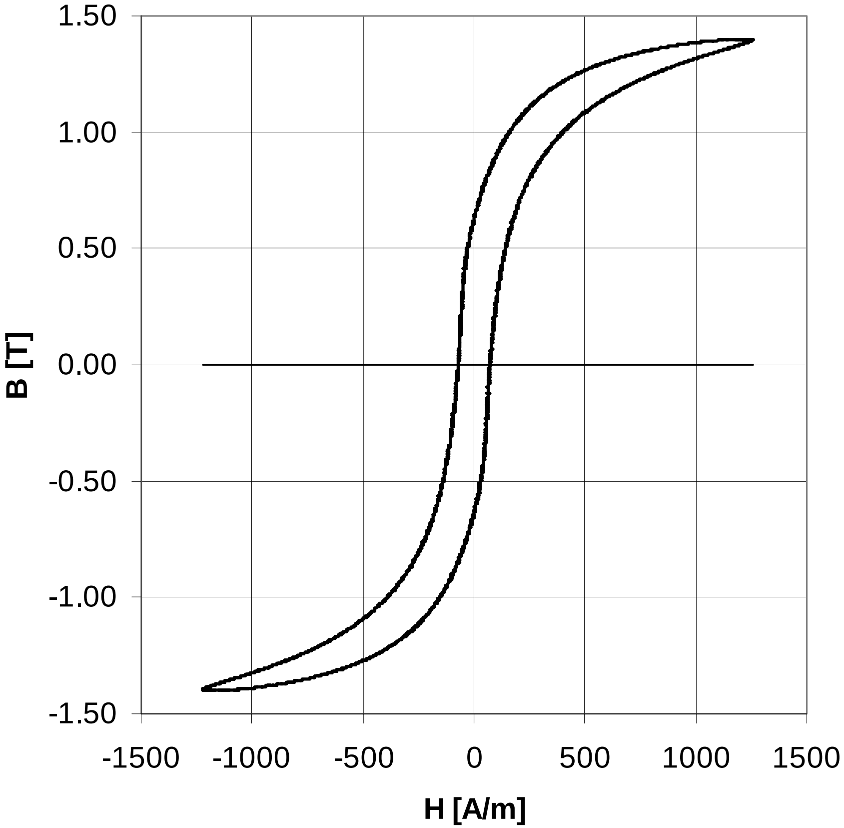
Additional equation: $\nabla \cdot \boldsymbol{J} = 0$ or $$\nabla \cdot \left(\sigma \frac{\partial \boldsymbol{A}}{\partial t} + \sigma \nabla \phi \right) = 0$$

Continuity:

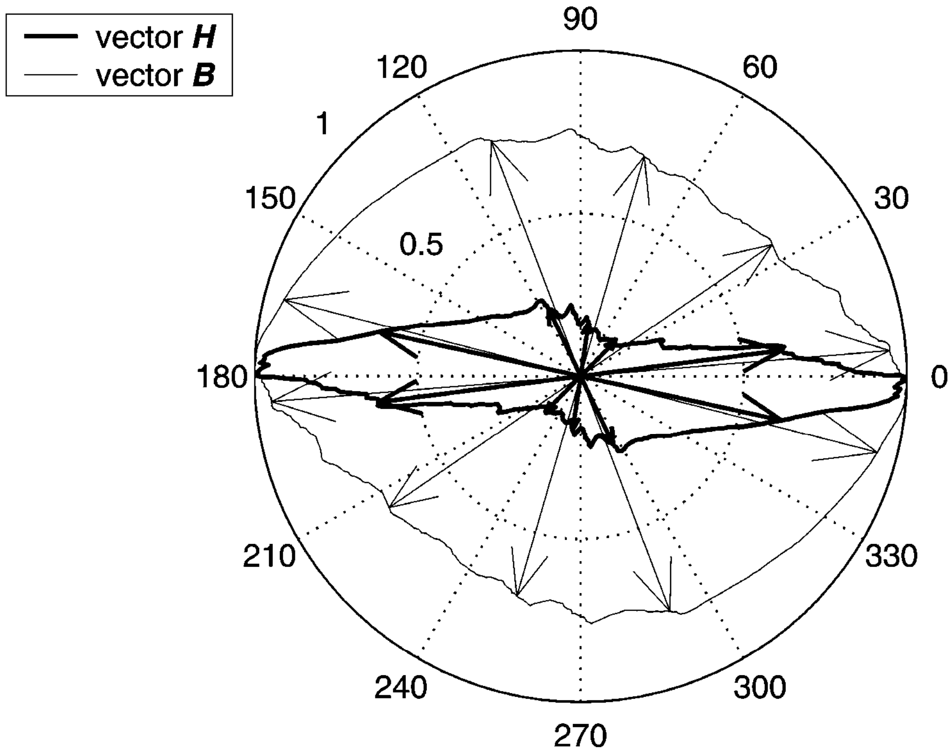
$$\left(\frac{\partial \boldsymbol{A}}{\partial t} + \nabla \phi \right) \cdot \boldsymbol{t}$$ is continuous $(\boldsymbol{E} \cdot \boldsymbol{t})$

$$\sigma \left(\frac{\partial \boldsymbol{A}}{\partial t} + \nabla \phi \right) \cdot \boldsymbol{n}$$ is continuous $(\boldsymbol{J} \cdot \boldsymbol{n})$

Magnetic characteristics of iron

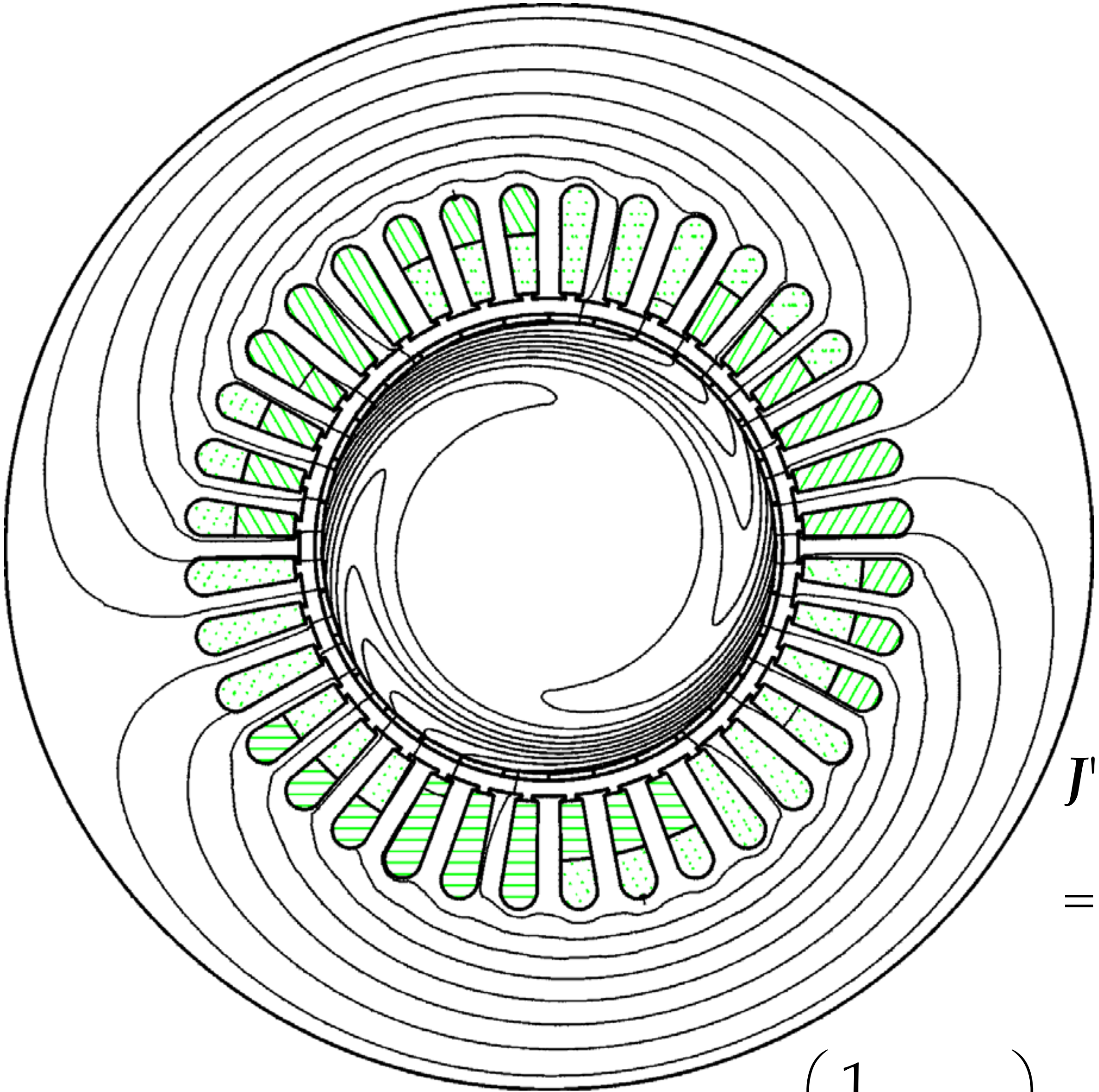


Alternating field



Field in an electrical machine

Modelling motion I



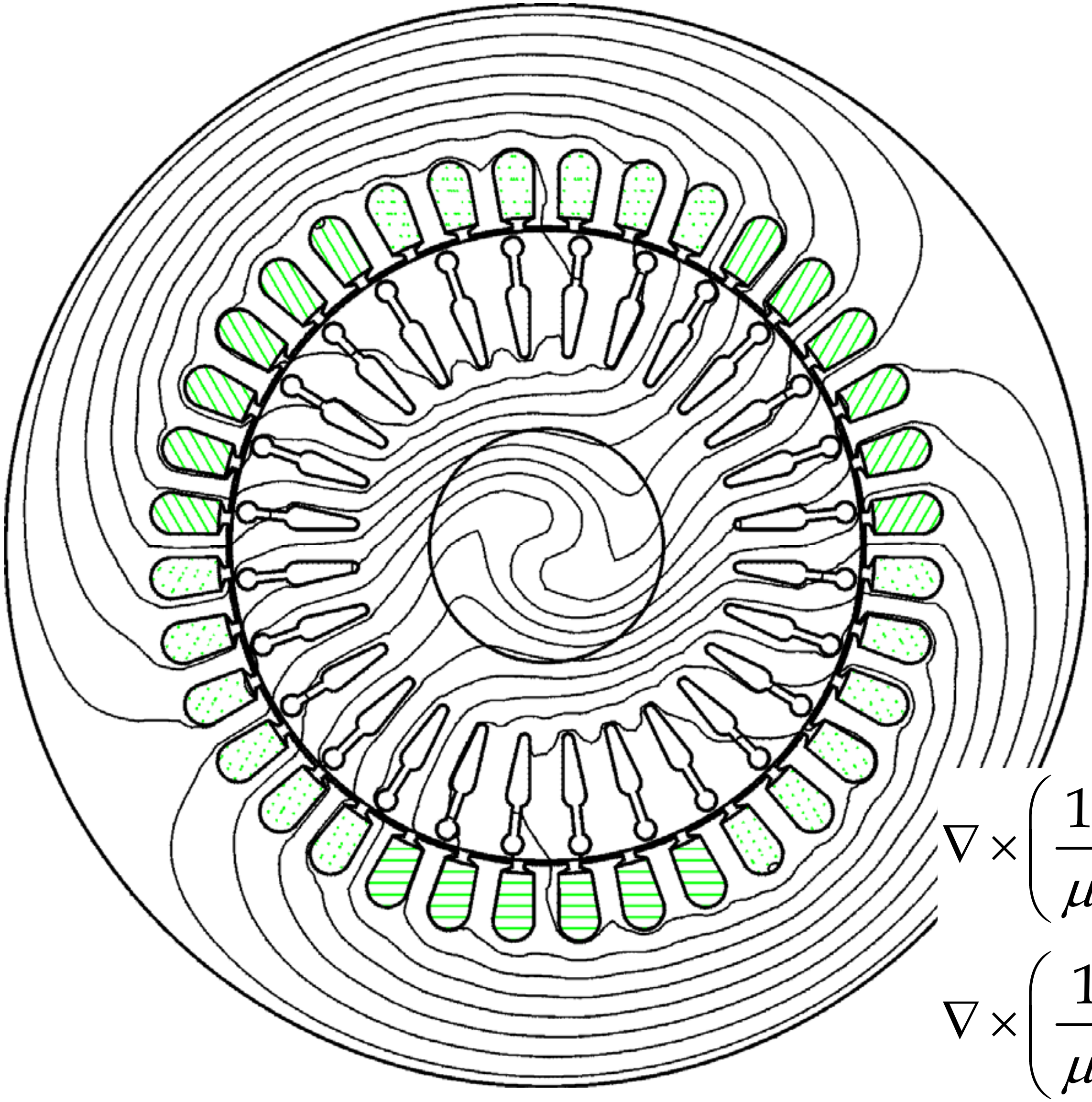
The rotor of a high-speed induction motor is a non-slotted, copper-coated steel cylinder. The motion does not affect the material properties observed from the stator. Both the stator and rotor fields can be solved in the stator frame of reference after adding the motion voltage term $\boldsymbol{v} \times \boldsymbol{B}$

$$\boldsymbol{J}' = \sigma \boldsymbol{E}' = \sigma \left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} \right)$$

$$= -\sigma \left(\frac{\partial \boldsymbol{A}}{\partial t} + \sigma \nabla \phi - \boldsymbol{v} \times \nabla \times \boldsymbol{A} \right)$$

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \boldsymbol{A} \right) + \sigma \left(\frac{\partial \boldsymbol{A}}{\partial t} - \boldsymbol{v} \times \nabla \times \boldsymbol{A} + \sigma \nabla \phi \right) = 0$$

Modelling motion II



Typically in electrical machines, there is a slotting in both the stator and rotor, and the material properties seen from any single frame of reference are time dependent.

The stator and rotor fields must be solved separately and forced to be continuous over the air gap.

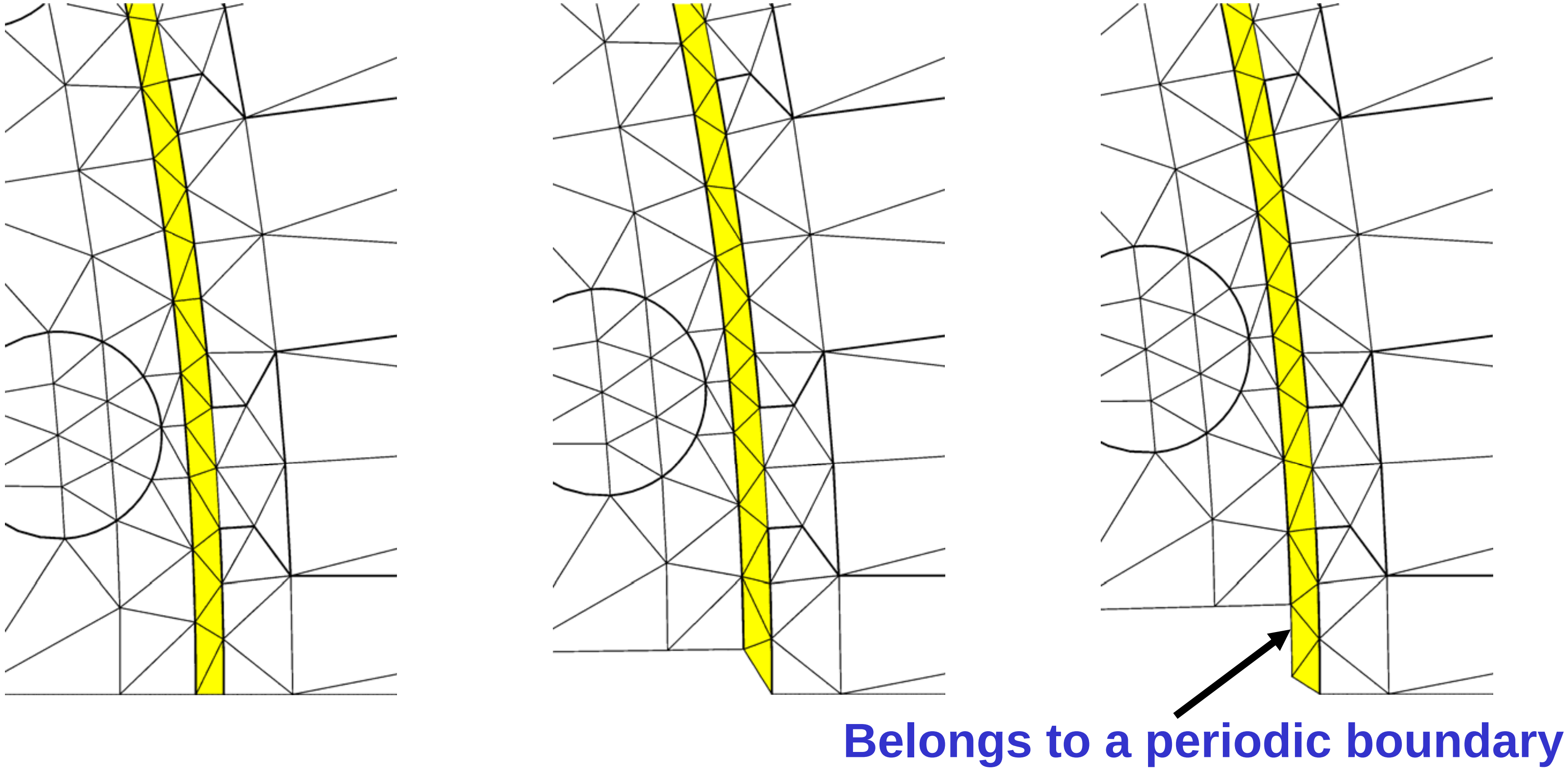
$$\nabla \times \left(\frac{1}{\mu} \nabla \times \boldsymbol{A} \right) + \sigma \frac{\partial \boldsymbol{A}}{\partial t} + \sigma \nabla \phi = 0$$

$$\nabla \times \left(\frac{1}{\mu'} \nabla \times \boldsymbol{A}' \right) + \sigma' \frac{\partial \boldsymbol{A}'}{\partial t} + \sigma' \nabla \phi' = 0$$

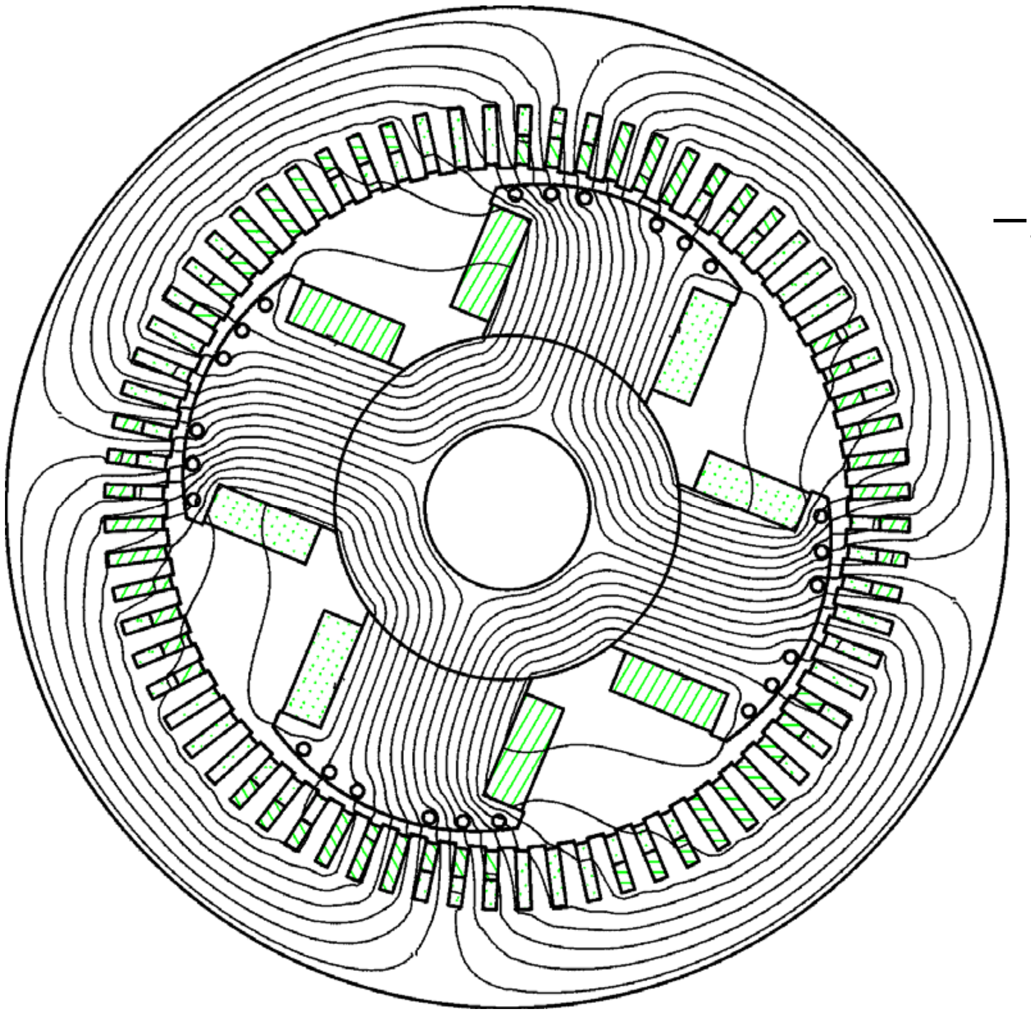
In air gap: $\boldsymbol{A} = \boldsymbol{A}'$

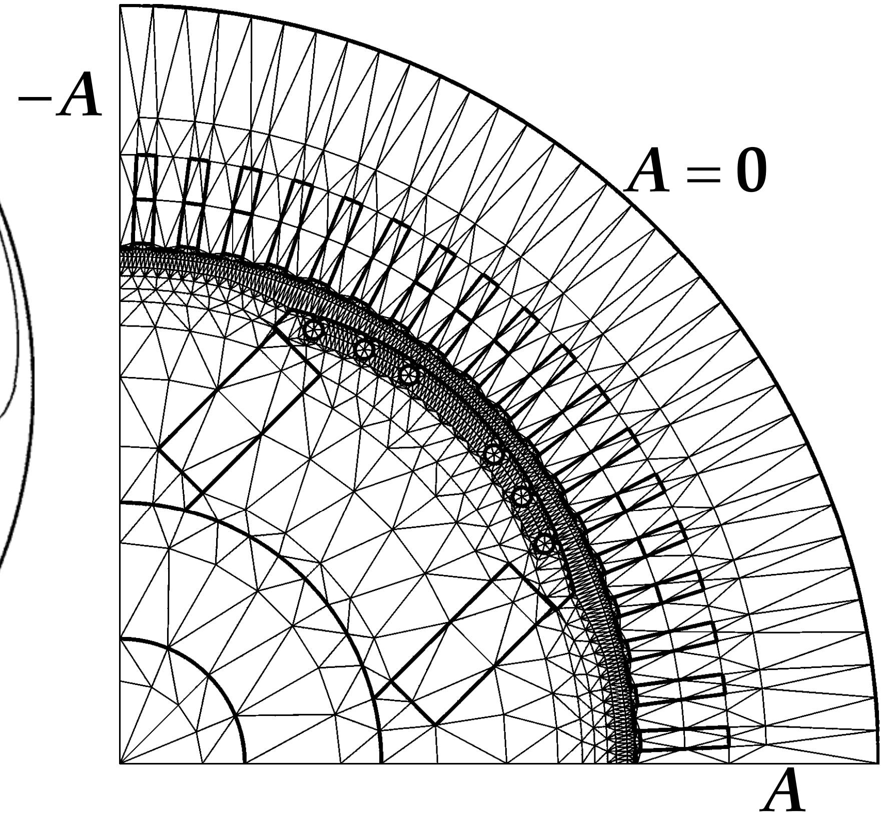
Rotation within FEA

The elements in the air gap are modified to allow continuous motion of the rotor. A typical time step in such a process is 30 – 50 μs, i.e. one period of line frequency is divided in 400 – 600 time steps.



Periodicity conditions for magnetic field





The geometry of an electrical machine typically repeats itself after one or two pole pitches.

Two-dimensional magnetic field

Current density and vector potential point in the same direction

$$\begin{cases} \boldsymbol{J} = J(x,y)\mathbf{e}_z \\ \boldsymbol{A} = A(x,y)\mathbf{e}_z \end{cases}$$

According to the definition, the flux density is

$$\boldsymbol{B} = B_x\mathbf{e}_x + B_y\mathbf{e}_y = \nabla\times\boldsymbol{A} = \nabla\times\left[A(x,y)\mathbf{e}_z\right]$$

$$\Rightarrow \quad B_x = \frac{\partial A}{\partial y} \qquad\qquad B_y = -\frac{\partial A}{\partial x}$$

Two-dimensional magnetic field II

Partial differential equation for a 2D vector potential

$$-\left[\frac{\partial}{\partial x}\left(\nu\frac{\partial \mathrm{A}}{\partial \mathrm{x}}\right)+\frac{\partial}{\partial y}\left(\nu\frac{\partial \mathrm{A}}{\partial y}\right)\right]\mathbf{e}_{\mathrm{z}} = J\mathbf{e}_{\mathrm{z}} \qquad \text{or} \qquad \nabla\cdot(\nu\nabla A) = -J$$

Similar equation as for the scalar potential of electric field, but the **boundary conditions have a different meaning.**

Dirichlet's condition (A is constant) means that the field is parallel to the boundary.

Homogenous Neumann's condition $\nu\frac{\partial A}{\partial n} = 0$ means that the field is perpendicular to the boundary.

Coupling circuit equations to field solution

Circuit equations for windings are

$$u_i = R_i i_i + L_i \frac{\mathrm{d}i_i}{\mathrm{d}t} + \frac{\mathrm{d}\Psi_i}{\mathrm{d}t}, \qquad i = 1,\ldots,m$$

R_i is resistance of a winding
L_i is series inductance
Ψ_i is flux linkage computed using FEM:

$$\Psi_i = \sum_{j=1}^{n} G_{ij} a_j, \qquad i = 1,\ldots,m$$

$$G_{ij} = \frac{K_i l}{C_{\mathrm{T}i}} \int_{\Omega} \beta_i N_j \, \mathrm{d}\Omega, \qquad i = 1,\ldots,m, \quad j = 1,\ldots,n$$

Coupling circuit equations to field solution II

Voltage equations in matrix form

$$\boldsymbol{u} = \boldsymbol{R}\boldsymbol{i} + \boldsymbol{L}\left\{\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}\right\} + \boldsymbol{G}\left\{\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{a}\right\}$$

The field and circuit equations are solved together

$$\begin{cases} \boldsymbol{S}(\boldsymbol{a})\boldsymbol{a} + \boldsymbol{T}\left\{\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{a}\right\} - \boldsymbol{F}\boldsymbol{i} = 0 \\ \boldsymbol{G}\left\{\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{a}\right\} + \boldsymbol{R}\boldsymbol{i} + \boldsymbol{L}\left\{\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}\right\} = \boldsymbol{u} \end{cases}$$

This can be written as one matrix equation

$$\boxed{\begin{bmatrix} \boldsymbol{S}(\boldsymbol{a}) & -\boldsymbol{F} \\ 0 & \boldsymbol{R} \end{bmatrix} \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{i} \end{bmatrix} + \begin{bmatrix} \boldsymbol{T} & 0 \\ \boldsymbol{G} & \boldsymbol{L} \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{i} \end{bmatrix} = \begin{bmatrix} 0 \\ \boldsymbol{u} \end{bmatrix}}$$

Coupling circuit equations to field solution III

Time discretisation $t^{k+1} = t^k + \Delta t$ by Crank-Nicolson method leads to a system of equations

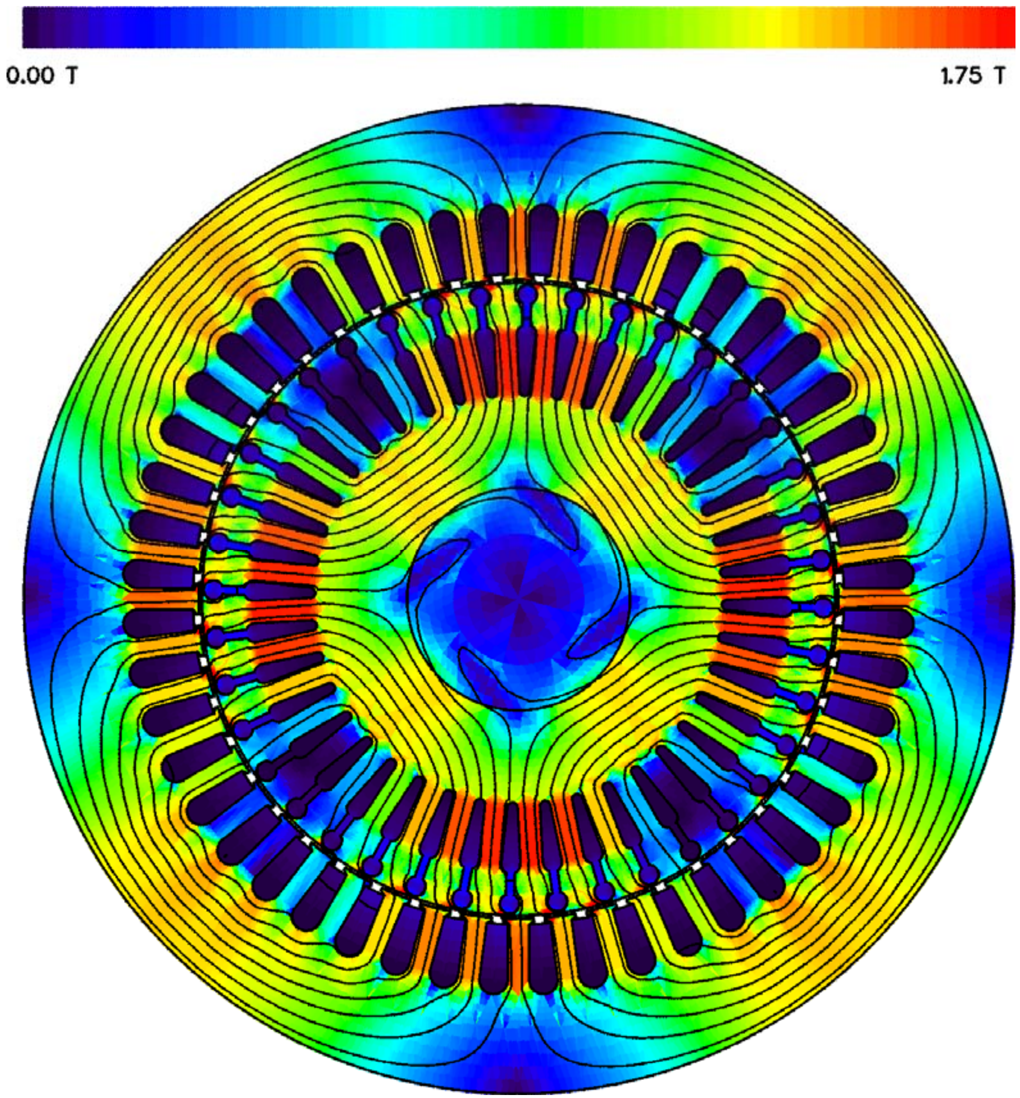
$$\begin{bmatrix} \boldsymbol{S}\left(\boldsymbol{a}^{k+1}\right) + \frac{2}{\Delta t}\boldsymbol{T} & -\boldsymbol{F} \\ \frac{2}{\Delta t}\boldsymbol{G} & \boldsymbol{R} + \frac{2}{\Delta t}\boldsymbol{L} \end{bmatrix} \begin{bmatrix} \boldsymbol{a}^{k+1} \\ \boldsymbol{i}^{k+1} \end{bmatrix} =$$

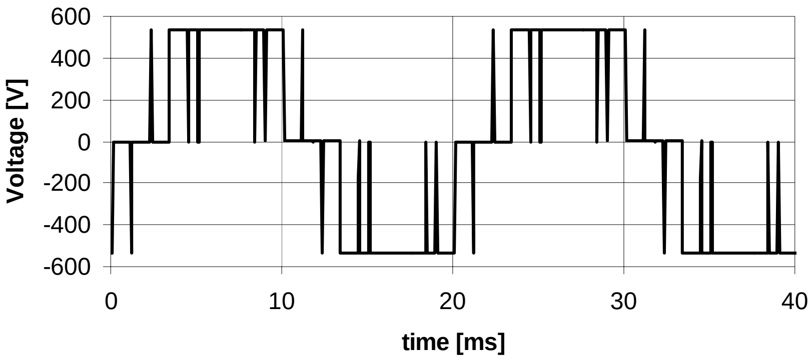
$$- \begin{bmatrix} \boldsymbol{S}\left(\boldsymbol{a}^{k}\right) - \frac{2}{\Delta t}\boldsymbol{T} & -\boldsymbol{F} \\ -\frac{2}{\Delta t}\boldsymbol{G} & \boldsymbol{R} - \frac{2}{\Delta t}\boldsymbol{L} \end{bmatrix} \begin{bmatrix} \boldsymbol{a}^{k} \\ \boldsymbol{i}^{k} \end{bmatrix} + \begin{bmatrix} 0 \\ \boldsymbol{u}^{k} \end{bmatrix} + \begin{bmatrix} 0 \\ \boldsymbol{u}^{k+1} \end{bmatrix}$$

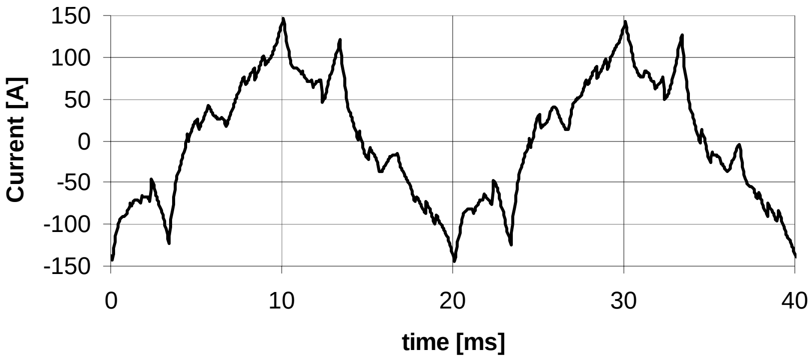
All the terms on the right hand side are known. If a sinusoidal time variation is assumed, the system of equations is

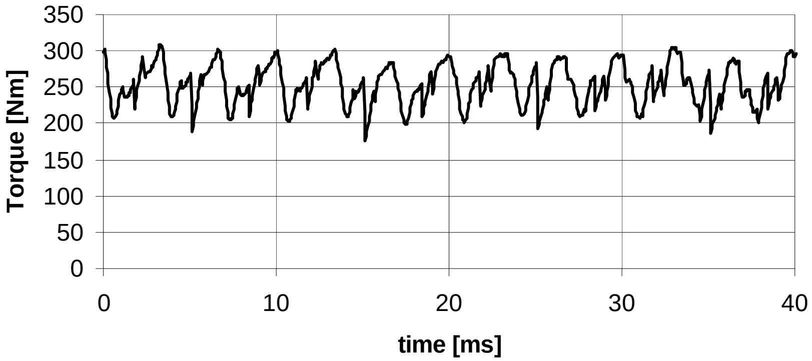
$$\begin{bmatrix} \boldsymbol{S}(\underline{\boldsymbol{a}}) & -\boldsymbol{F} \\ 0 & \boldsymbol{R} \end{bmatrix} \begin{bmatrix} \underline{\boldsymbol{a}} \\ \underline{\boldsymbol{i}} \end{bmatrix} + \mathrm{j}\omega \begin{bmatrix} \boldsymbol{T} & 0 \\ \boldsymbol{G} & \boldsymbol{L} \end{bmatrix} \begin{bmatrix} \underline{\boldsymbol{a}} \\ \underline{\boldsymbol{i}} \end{bmatrix} = \begin{bmatrix} 0 \\ \underline{\boldsymbol{u}} \end{bmatrix}$$

Electromagnetic analysis



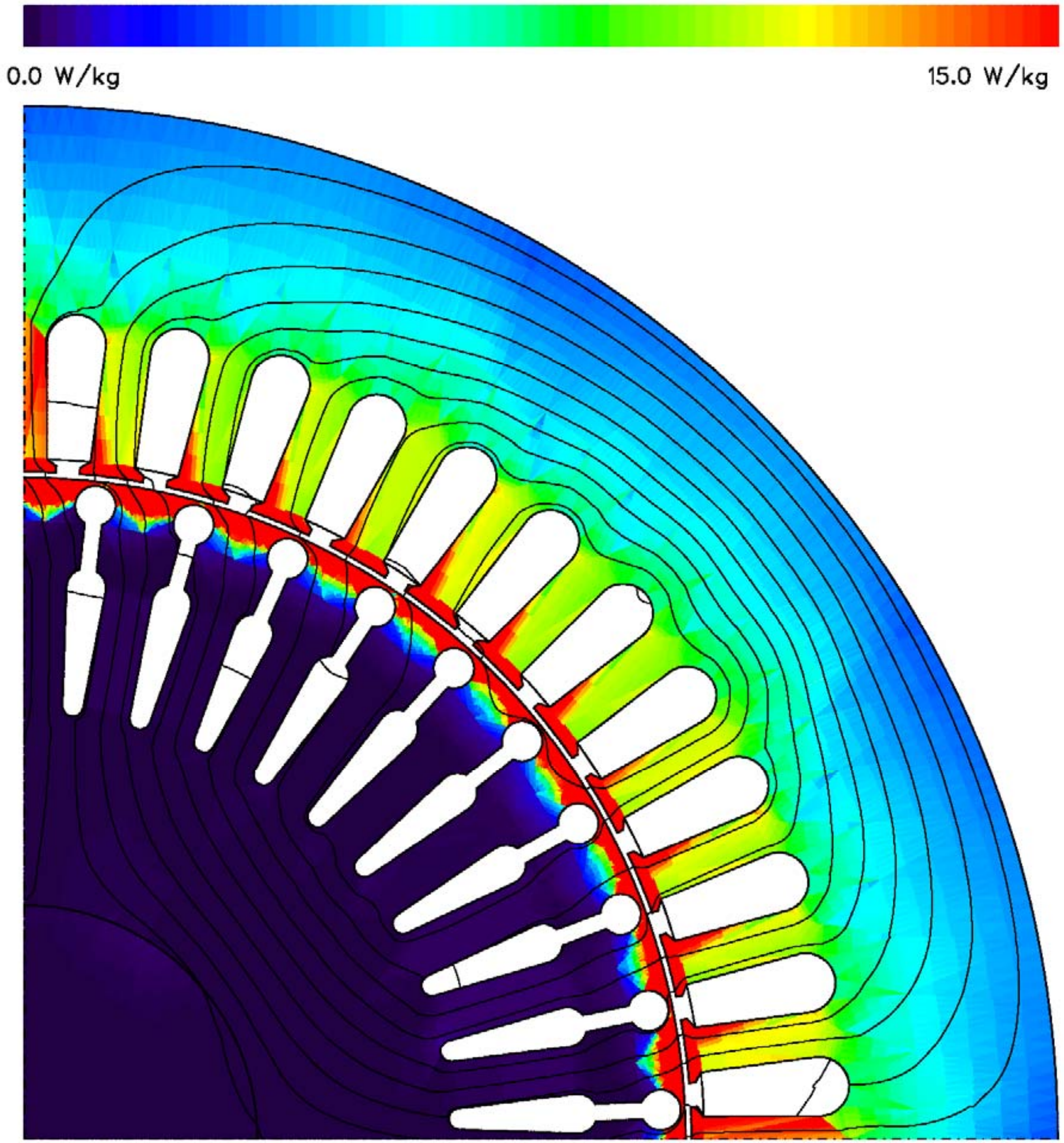




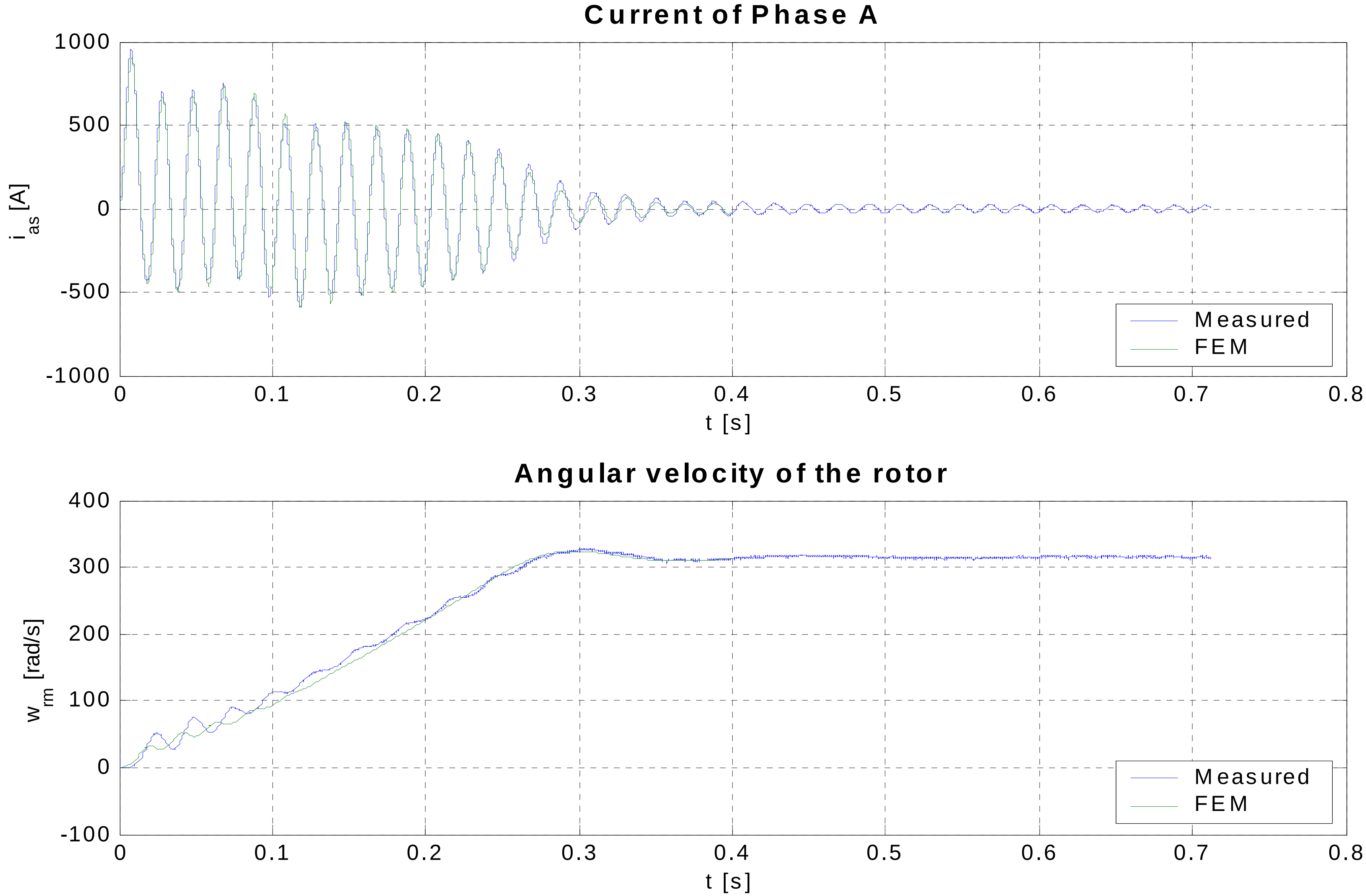


Loss distribution in a core

Time-discretised finite element analysis combined with a dynamic hysteresis model has been used to study an inverter-fed cage induction motor.

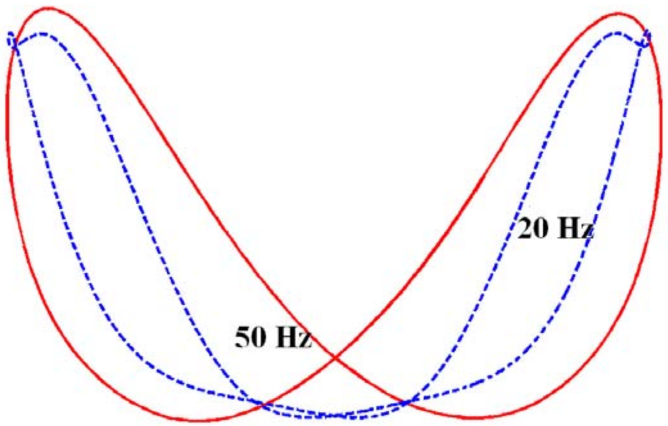


Validation – Starting of the two-pole 30 kW motor



Magneto-mechanical characteristics of electrical steels

Aim – to develop a magneto-mechanical coupled model for electrical machines and study magnetostriction in electrical steel sheets



PhD thesis in pre-examination

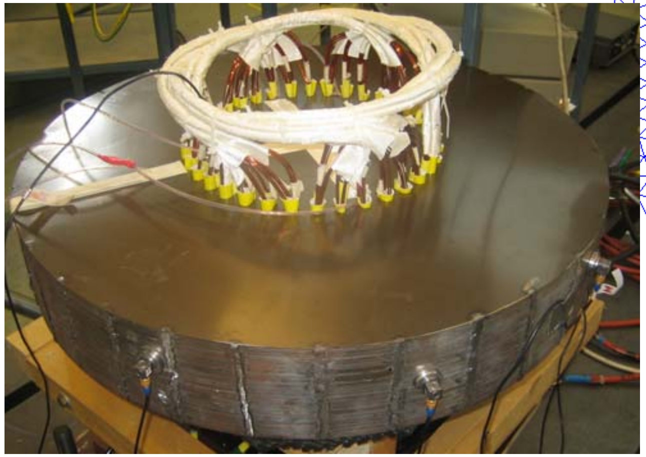
Katarzyna FONTEYN

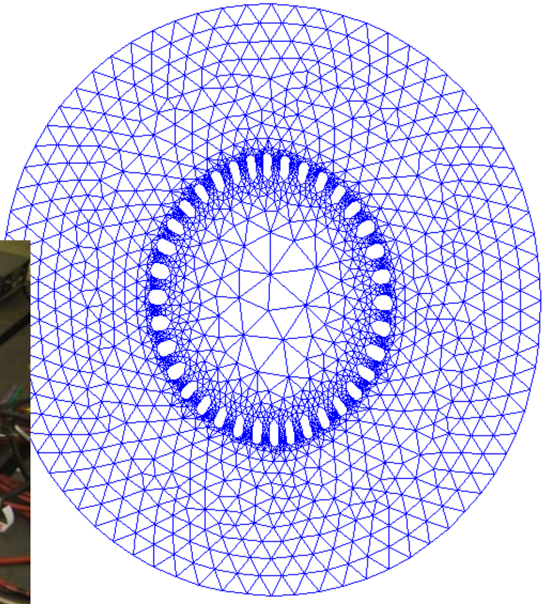
Funding

TEKES, TKK

Collaboration

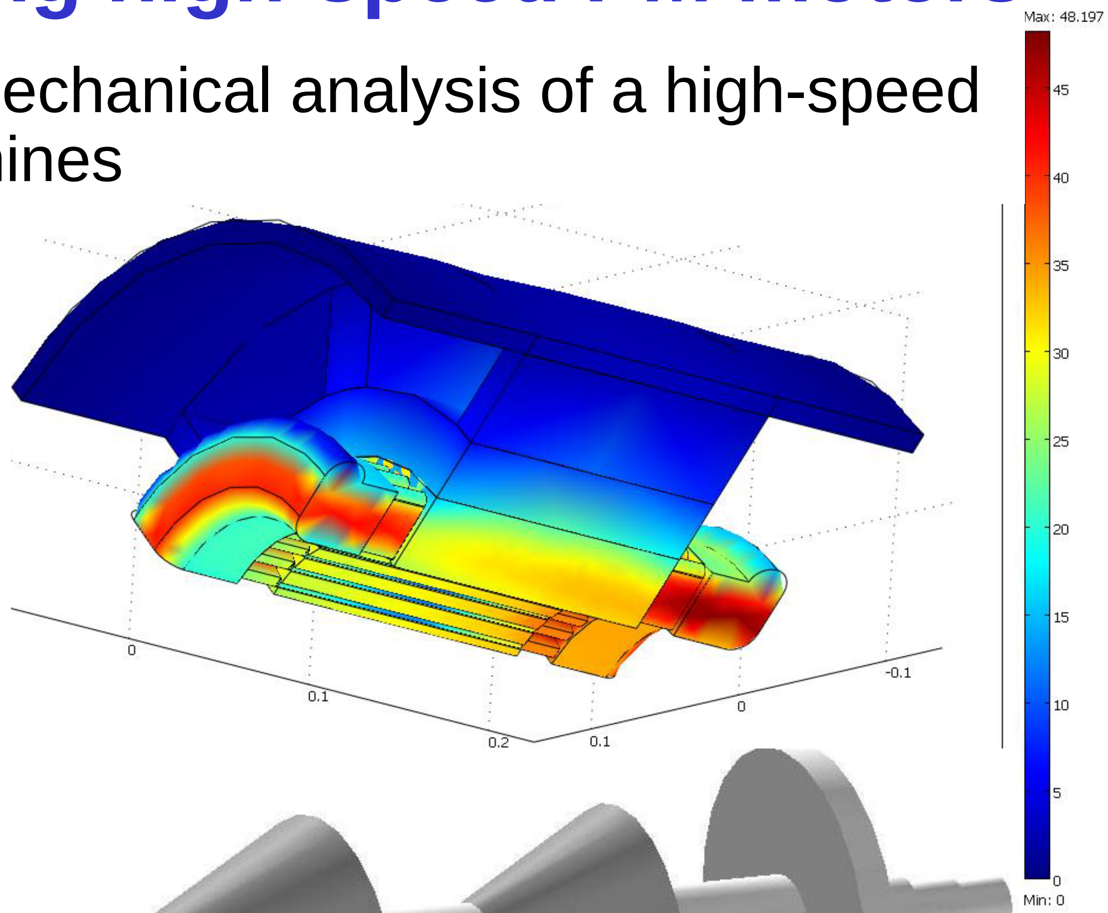
Several units from TKK and TTY





Tools for analysing high-speed PM motors

Aim – tools for thermo-mechanical analysis of a high-speed permanent-magnet machines

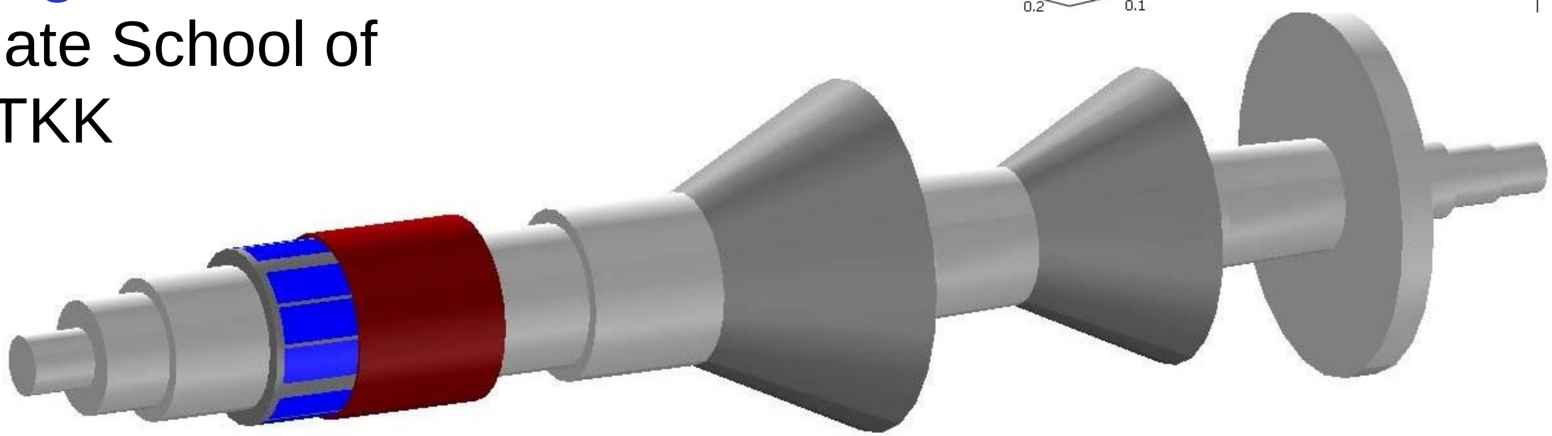


PhD thesis in pre-examination

Zlatko KOLONDZOVSKI

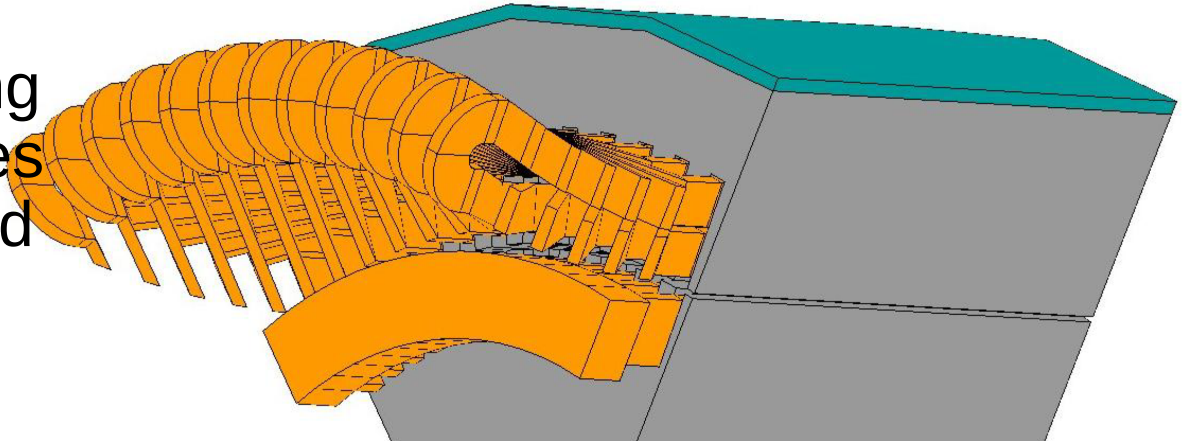
Funding

Graduate School of EEE, TKK



Collaboration: LTY / Laboratory of Fluid Dynamics

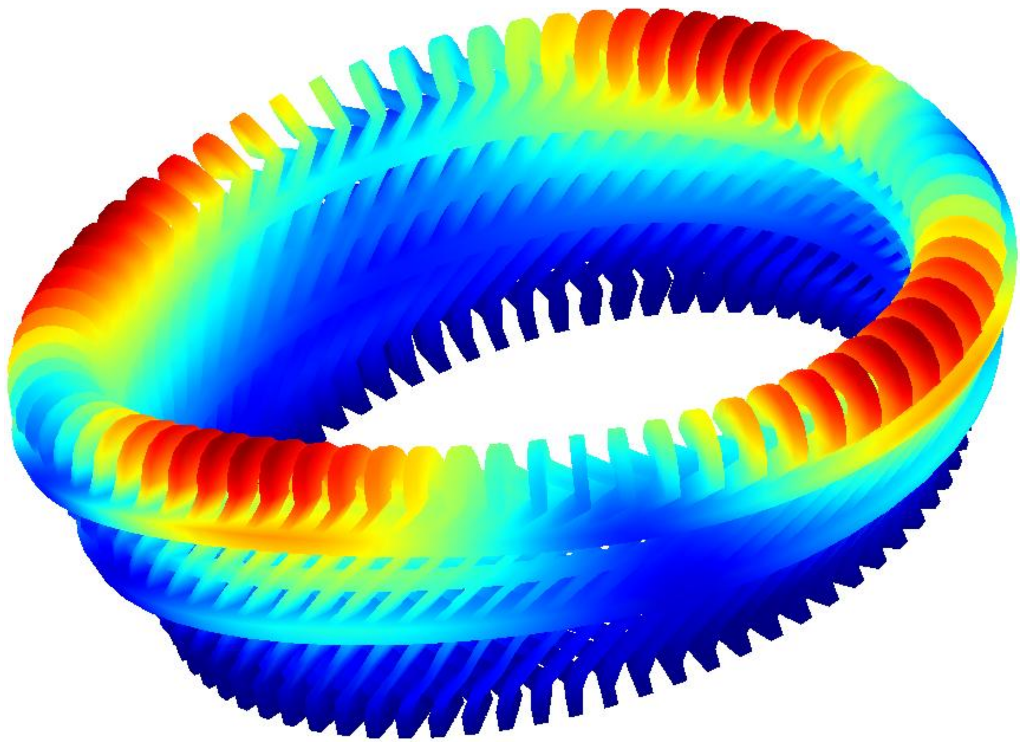
End-winding fields, forces and vibrations



Aim – tools for analysing the magnetic field, forces and vibrations at the end windings of electrical machines.

PhD thesis in pre-examination

Ranran LIN



Funding

The Finnish Technology Award Foundation,

TKK