TKK Matematiikan laitos Arponen/Pursiainen

Mat-1.3651 Numerical Linear Algebra, spring 2008
(Numeerinen matriisilaskenta, kevät 2008)

Exercise 3 (7.2.2008)

Please hand in the exercises marked with an asterisk (*) either to the assistant's folder in front of U313 or latest at the beginning of the exercise.

* 1. Using the SVD, prove that any matrix in $\mathbb{C}^{m\times n}$ is the limit of a sequence of matrices of full rank. In other words, prove that the set of full-rank matrices is a dense subset of $\mathbb{C}^{m\times n}$. Use the 2-norm for your proof.

* 2.

(a) If P is an orthogonal projector, then $I - 2P$ is unitary. Prove this algebraically, and give a geometric interpretation.

(b) Given $A \in \mathbb{C}^{m\times n}$ with $m \geq n$, show that A^*A is nonsingular if and only if A has full rank.

3. Let E be the $m \times m$ matrix that extracts the "even part" of an m-vector: $Ex = (x + Fx)/2$ where F is the $m \times m$ matrix that flips $(x_1, \ldots, x_m)^*$ to $(x_m, \ldots, x_1)^*$. Is E a projector? If yes, is it an orthogonal projector? What are its entries?

* 4. Consider the matrices

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

(a) What is the orthogonal projector P onto $\mathcal{R}(A)$ (the range of A), and what is the image under P of the vector $v = (1, 2, 3)^*$?

(b) Same questions for B.

5. Let $P \in \mathbb{C}^{m\times m}$ be a nonzero projector. Show that $\|P\|_2 \geq 1$, with equality if and only if P is an orthogonal projector.

* 6. Let A be an $m \times n$ matrix ($m \geq n$) of full rank with the property that columns $1, 3, 5, \ldots$ are orthogonal to columns $2, 4, 6, \ldots$. In a reduced QR factorization $A = \hat{Q}\hat{R}$, what special structure does $\hat{R}$ possess?