FUNCTIONAL ITO FORMULA AND STOCHASTIC INTEGRAL REPRESENTATION FOR FUNCTIONALS OF SEMIMARTINGALES

RAMA CONT & DAVID FOURNIE

Abstract

We present a *functional* extension of the Ito formula which allows to obtain explicit stochastic integral representations for non-anticipative functionals of continuous semimartingales with continuous dependence on the underlying path and its quadratic variation. Our formula extends recent work by B. Dupire to functionals which can depend on quadratic variation and allows for a natural interpretation in financial applications in terms of sensitivities. As a by-product, we show that a large class of functionals of a Brownian semimartingale can be described as solutions of a pathwise functional heat equation. This leads to model-free relations between sensitivities of path-dependent options.

CNRS - Université de Paris VI & Columbia University, New York
E-mail address: Rama.Cont@upmc.fr

Date: Non-Semimartingale Techniques in Mathematical Finance
May 26-28, 2009, Helsinki University of Technology .