APPROXIMATING A GEOMETRIC FRACTIONAL BROWNIAN MOTION AND RELATED PROCESSES VIA DISCRETE WICK CALCULUS

C. BENDER

Abstract

The stochastic exponential $e^{B_t^H - t^{2H}/2}$ of a fractional Brownian motion B^H is intimately connected to the Wick product. On the one hand it admits a series representation in terms of Wick powers. On the other hand it solves a Doleans-Dade type stochastic differential equation with the integral interpreted in the fractional Wick-Itô-Skorokhod sense. We replace the fractional Brownian motion B^H by a weakly convergent approximation based on binary trials [3] and the Wick product by its rather simple discrete counterpart acting on the binary trials [2]. In this way we obtain approximations of $e^{B_t^H - t^{2H}/2}$ in terms of a discrete Wick power series and a discrete Wick difference equation. Weak convergence of both approximations is proved. Moreover, the approach is extended in order to establish weak Euler schemes for some systems of linear SDEs driven by a fractional Brownian motion in the fractional Wick-Itô-Skorokhod sense. In particular, we derive weak convergence of the discrete version of the fractional Black-Scholes market [1].

The talk is based on a joint work with Peter Parczewski (TU Braunschweig).

References

[1] Bender, C. and Elliott, R. J. (2004) Arbitrage in a discrete version of the Wick-fractional Black-Scholes market. *Math. Oper. Res.* **29**, 935-945.

[2] Holden, H. and Lindstrøm, T. and Øksendal, B. and Ubøe, J. (1993) Discrete Wick products. In: Lindstrøm, T. (ed.) et al., *Stochastic analysis and related topics.* Yverdon: Gordon and Breach, p. 123-148.

[3] Sottinen, Tommi (2001) Fractional Brownian motion, random walks and binary market models. *Finance Stoch.* **5**, (2001), 343-355.

Saarland University
E-mail address: c.bender@tu-bs.de