Exercise 6
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1 Show that if $F = \text{e-lim}\, F^\nu$ and G is continuous, then

$$\text{e-lim}(F^\nu + G) = F + G.$$

2 A sequence $(C^\nu)_{\nu=1}^\infty$ of sets in a topological space is said to converge to a set C in the sense of Painlevé-Kuratowski if

(i) if $x^\nu \to x$ and $x^{\nu^\mu} \in C^{\nu^\mu}$ for some subsequence, then $x \in C$,

(ii) for every $x \in C$ there is a sequence $x^\nu \to x$ with $x^\nu \in C^\nu$.

Show that

(a) $C^\nu \to C$ in the sense of Painlevé-Kuratowski iff $\text{e-lim}\, \delta_{C^\nu} = \delta_C$.

(b) $\text{e-lim}\, f^\nu = f$ iff $\operatorname{epi} f^\nu \to \operatorname{epi} f$ in the sense of Painlevé-Kuratowski.

3 Let $\tilde{F}^\nu : (\mathbb{R}^n)^{I(\nu)} \to \overline{\mathbb{R}}$ be the essential objective of (3.3). Show that, if condition (3.2) holds, then

$$F^\nu(\Pi^\nu z^\nu) = \tilde{F}^\nu(z^\nu),$$

where $\Pi^\nu : (\mathbb{R}^n)^{I(\nu)} \to L^p$ is given by

$$\Pi^\nu z^\nu = \sum_{i \in I(\nu)} z^{\nu,i} \chi^{\nu,i}.$$

4 Show that if $P(\Xi^{\nu,i}) > 0$ for all $i \in I(\nu)$, then the set $\mathcal{N}^\nu$ is isomorphic to

$$\tilde{\mathcal{N}}^\nu = \{x \in L^p(\Xi, \mathcal{F}, P^\nu; \mathbb{R}^n) \,|\, x \text{ contains an } (\mathcal{F}_k)_{k=0}^K\text{-adapted function}\}.$$

What metric on $\mathcal{N}^\nu$ would make them isometric?

5 Consider the mapping s^ν defined in Section 3.2 and show that $f^\nu(x,\xi) = f(x, s^\nu(\xi))$ is a normal integrand whenever f is one.

6 Prove Lemma 3.9.