Helsinki University of Technology Somersalo/Bingham
Mathematics

Mat-1.198 Scattering Theory

3rd set of exercises, 12.2.2003

N.B. There is no lecture on Monday, the 10th of February.

1. Let ∂D be a bounded C^2 surface. Using the estimate
$$|n(y)\cdot(z-y)| \leq L|z-y|^2 \qquad \text{for all } z,y\in\partial D,$$
show that there is an $h_0>0$ such that in
$$B=\left\{x=z+hn(z)\,|\,z\in\partial D,\,|h|\leq h_0\right\}$$
the *boundary projection* $z+hn(z)\mapsto z$ is unique. Further, show that it is possible to choose h_0 so small that
$$|x_1-x_2|^2 \geq \frac{1}{2}|z_1-z_2|^2 \qquad \text{for all } x_1,x_2\in B,$$
where z_j is the boundary projection of x_j.

2. Show that for $0<\beta<1$, we have the estimate
$$\left|t\ln\frac{1}{t}\right| \leq \frac{1}{1-\beta}t^{\beta} \qquad \text{when } 0<t<1.$$

3. *(Jump relation, special case)* Let
$$\Phi_0(x)=\frac{1}{4\pi|x|}$$
and define
$$w(x)=\int_{\partial D}\frac{\partial\Phi_0}{\partial n(y)}(x-y)\,dS(y), \qquad x\notin\partial D$$
where ∂D is a bounded C^2 surface. Prove that
$$w(x)=\begin{cases}-1, & x\in D\\ 0, & x\in\mathbb{R}^3\setminus\overline{D}.\end{cases}$$

4. *(Jump relation, continued)* Show that
$$w(x)=-\frac{1}{2} \qquad \text{when } x\in\partial D$$
and w is interpreted as an improper integral.