Mat-1.3608 Markov chains. Notation: $s_\mu(n) = s(\mu P^n; \pi)$ where P is a transition matrix and π is a stationary distribution.

VI Exercise 28.2. 2008 Tikanmäki/Valkeila.

1. Show that $s_\mu(n) \geq s_\mu(n+k)$ for $n \geq 0, k \geq 0$ for any transition matrix P and stationary distribution π.
2. Give and example of P and π such that $s_\mu(n) = c$ for all $n \geq 1$, where $c > 0$.
3. Let $X = (X_n)_{n \geq 0}$ be a (P, π) Markov chain, where π is the stationary distribution and also the initial distribution. Define Q by $\pi_i q_{ij} = \pi_j p_{ji}$ and extend X to the negative side as follows:

$$\mathbb{P}(X_{-1} = s_{i_1}, X_{-2} = s_{i_2}, \dots X_{-k} = s_{i_k} | X_0 = s_j, X_1 = s_{j_1}, \dots X_n = s_{j_n}) =$$
$$\mathbb{P}(X_{-1} = s_{i_1}, X_{-2} = s_{i_2}, \dots X_{-k} = s_{i_k} | X_0 = s_j) = q_{ji_1} q_{i_1 i_2} \cdots q_{i_{k-1} i_k}$$

   for all $k \geq 1$ and $n \geq 1$. Show that $(X_n)_{-\infty < n < \infty}$ is a Markov chain with transition matrix P and $\mathbb{P}(X_n = s_i) = \pi_i$ for all n and $s_i \in S$. [This clarifies the proof of Theorem 5.4, p.223 in Brémaud.]

4. Let α, β be two probability distributions on S, let Y be a random variable with distribution α and Z is a random variable with distribution β. Show that
$$d_V(\alpha, \beta) \leq \mathbb{P}(Y \neq Z).$$
5. Let α, β be two probability measures on S and define $\nu(\alpha, \beta)$ by
$$\nu(\alpha, \beta) = \sum_j \min(\alpha_j, \beta_j).$$
   Show that
$$d_V(\alpha, \beta) = 1 - \nu(\alpha, \beta).$$
6. Please go to Opinionsonline and make your comments on the course.