CONTENTS

Part A. Ordinary Differential Equations. 1

Chapter 1 First-Order Differential Equations. 2

1.1 Basic Concepts and Ideas, 2
1.2 Geometrical Meaning of $y' = f(x, y)$. Direction Fields, 10
1.3 Separable Differential Equations, 14
1.4 Modeling: Separable Equations, 19
1.5 Exact Differential Equations. Integrating Factors, 25
1.6 Linear Differential Equations. Bernoulli Equation, 33
1.7 Modeling: Electric Circuits, 41
1.8 Orthogonal Trajectories of Curves. *Optional,* 48
1.9 Existence and Uniqueness of Solutions. Picard Iteration, 52
Chapter Review, 59
Chapter Summary, 61

Chapter 2 Linear Differential Equations of Second and Higher Order 64

2.1 Homogeneous Linear Equations of Second Order, 64
2.2 Second-Order Homogeneous Equations with Constant Coefficients, 72
2.3 Case of Complex Roots. Complex Exponential Function, 76
2.4 Differential Operators. *Optional,* 81
2.5 Modeling: Free Oscillations (Mass–Spring System), 83
2.6 Euler–Cauchy Equation, 93
2.7 Existence and Uniqueness Theory. Wronskian, 97
2.8 Nonhomogeneous Equations, 101
2.9 Solution by Undetermined Coefficients, 104
2.10 Solution by Variation of Parameters, 108
2.11 Modeling: Forced Oscillations. Resonance, 111
2.12 Modeling of Electric Circuits, 118
2.13 Higher Order Linear Differential Equations, 124
2.14 Higher Order Homogeneous Equations with Constant Coefficients, 132
2.15 Higher Order Nonhomogeneous Equations, 138
Chapter Review, 142
Chapter Summary, 143

Chapter 3 Systems of Differential Equations, Phase Plane, Qualitative Methods 146

3.0 Introduction: Vectors, Matrices, Eigenvalues, 146
3.1 Introductory Examples, 152
3.2 Basic Concepts and Theory, 159
3.3 Homogeneous Systems with Constant Coefficients. Phase Plane, Critical Points, 162
3.4 Criteria for Critical Points. Stability, 170
3.5 Qualitative Methods for Nonlinear Systems, 175
3.6 Nonhomogeneous Linear Systems, 184
Chapter Review, 190
Chapter Summary, 192

Chapter 4 Series Solutions of Differential Equations. Special Functions 194
4.1 Power Series Method, 194
4.2 Theory of the Power Series Method, 198
4.3 Legendre's Equation. Legendre Polynomials $P_n(x)$, 205
4.4 Frobenius Method, 211
4.5 Bessel's Equation. Bessel Functions $J_\nu(x)$, 218
4.6 Bessel Functions of the Second Kind $Y_\nu(x)$, 228
4.7 Sturm–Liouville Problems. Orthogonal Functions, 233
4.8 Orthogonal Eigenfunction Expansions, 240
Chapter Review, 247
Chapter Summary, 248

Chapter 5 Laplace Transforms 250
5.1 Laplace Transform. Inverse Transform. Linearity. Shifting, 251
5.2 Transforms of Derivatives and Integrals. Differential Equations, 258
5.3 Unit Step Function. Second Shifting Theorem. Dirac's Delta Function, 265
5.4 Differentiation and Integration of Transforms, 275
5.5 Convolution. Integral Equations, 279
5.6 Partial Fractions. Differential Equations, 284
5.7 Systems of Differential Equations, 291
5.8 Laplace Transform: General Formulas, 296
5.9 Table of Laplace Transforms, 297
Chapter Review, 299
Chapter Summary, 302

Part B. Linear Algebra, Vector Calculus. 303

Chapter 6 Linear Algebra: Matrices, Vectors, Determinants. Linear Systems of Equations 304
6.1 Basic Concepts. Matrix Addition, Scalar Multiplication, 305
6.2 Matrix Multiplication, 311
6.3 Linear Systems of Equations. Gauss Elimination, 321
6.4 Rank of a Matrix. Linear Independence. Vector Space, 331
6.5 Solutions of Linear Systems: Existence, Uniqueness, General Form, 338
6.6 Determinants. Cramer's Rule, 341
6.7 Inverse of a Matrix. Gauss–Jordan Elimination, 350
6.8 Vector Spaces, Inner Product Spaces, Linear Transformations. *Optional,* 358
Chapter Review, 365
Chapter Summary, 367

Chapter 7 Linear Algebra: Matrix Eigenvalue Problems 370
7.1 Eigenvalues, Eigenvectors, 371
7.2 Some Applications of Eigenvalue Problems, 376
7.3 Symmetric, Skew-Symmetric, and Orthogonal Matrices, 381
7.4 Complex Matrices: Hermitian, Skew-Hermitian, Unitary, 385
7.5 Similarity of Matrices. Basis of Eigenvectors. Diagonalization, 392
Chapter Review, 398
Chapter Summary, 399

Chapter 8 Vector Differential Calculus. Grad, Div, Curl. 400
8.1 Vector Algebra in 2-Space and 3-Space, 401
8.2 Inner Product (Dot Product), 408

8.3 Vector Product (Cross Product), 414
8.4 Vector and Scalar Functions and Fields. Derivatives, 423
8.5 Curves. Tangents. Arc Length, 428
8.6 Curves in Mechanics. Velocity and Acceleration, 435
8.7 Curvature and Torsion of a Curve. *Optional,* 440
8.8 Review from Calculus in Several Variables. *Optional,* 443
8.9 Gradient of a Scalar Field. Directional Derivative, 446
8.10 Divergence of a Vector Field, 453
8.11 Curl of a Vector Field, 457
Chapter Review, 459
Chapter Summary, 461

Chapter 9 Vector Integral Calculus. Integral Theorems 464
9.1 Line Integrals, 464
9.2 Line Integrals Independent of Path, 471
9.3 From Calculus: Double Integrals. *Optional,* 478
9.4 Green's Theorem in the Plane, 485
9.5 Surfaces for Surface Integrals, 491
9.6 Surface Integrals, 496
9.7 Triple Integrals. Divergence Theorem of Gauss, 505
9.8 Further Applications of the Divergence Theorem, 510
9.9 Stokes's Theorem, 515
Chapter Review, 521
Chapter Summary, 522

Part C. Fourier Analysis and Partial Differential Equations 525

Chapter 10 Fourier Series, Integrals, and Transforms 526
10.1 Periodic Functions. Trigonometric Series, 527
10.2 Fourier Series, 529
10.3 Functions of Any Period $p = 2L$, 537
10.4 Even and Odd Functions. Half-Range Expansions, 541
10.5 Complex Fourier Series. *Optional,* 547
10.6 Forced Oscillations, 550
10.7 Approximation by Trigonometric Polynomials, 553
10.8 Fourier Integrals, 557
10.9 Fourier Cosine and Sine Transforms, 564
10.10 Fourier Transform, 569
10.11 Tables of Transforms, 576
Chapter Review, 579
Chapter Summary, 580

Chapter 11 Partial Differential Equations. 582
11.1 Basic Concepts, 583
11.2 Modeling: Vibrating String, Wave Equation, 585
11.3 Separation of Variables. Use of Fourier Series, 587
11.4 D'Alembert's Solution of the Wave Equation, 595
11.5 Heat Equation: Solution by Fourier Series, 600
11.6 Heat Equation: Solution by Fourier Integrals and Transforms, 610
11.7 Modeling: Membrane, Two-Dimensional Wave Equation, 616
11.8 Rectangular Membrane. Use of Double Fourier Series, 619
11.9 Laplacian in Polar Coordinates, 626
11.10 Circular Membrane. Use of Fourier–Bessel Series, 629

11.11 Laplace's Equation in Cylindrical and Spherical Coordinates. Potential, 636
11.12 Solution by Laplace Transforms, 643
Chapter Review, 647
Chapter Summary, 648

Part D. Complex Analysis 651

Chapter 12 Complex Numbers and Functions. Conformal Mapping 652
12.1 Complex Numbers. Complex Plane, 652
12.2 Polar Form of Complex Numbers. Powers and Roots, 657
12.3 Derivative. Analytic Function, 663
12.4 Cauchy–Riemann Equations. Laplace's Equation, 669
12.5 Geometry of Analytic Functions: Conformal Mapping, 674
12.6 Exponential Function, 679
12.7 Trigonometric Functions, Hyperbolic Functions, 682
12.8 Logarithm. General Power, 687
12.9 Linear Fractional Transformations. *Optional,* 692
12.10 Riemann Surfaces. *Optional,* 699
Chapter Review, 701
Chapter Summary, 702

Chapter 13 Complex Integration 704
13.1 Line Integral in the Complex Plane, 704
13.2 Cauchy's Integral Theorem, 713
13.3 Cauchy's Integral Formula, 721
13.4 Derivatives of Analytic Functions, 725
Chapter Review, 730
Chapter Summary, 731

Chapter 14 Power Series, Taylor Series 732
14.1 Sequences, Series, Convergence Tests, 732
14.2 Power Series, 741
14.3 Functions Given by Power Series, 746
14.4 Taylor Series and Maclaurin Series, 751
14.5 Uniform Convergence. *Optional,* 759
Chapter Review, 767
Chapter Summary, 768

Chapter 15 Laurent Series, Residue Integration 770
15.1 Laurent Series, 770
15.2 Singularities and Zeros. Infinity, 776
15.3 Residue Integration Method, 781
15.4 Evaluation of Real Integrals, 787
Chapter Review, 794
Chapter Summary, 796

Chapter 16 Complex Analysis Applied to Potential Theory 798
16.1 Electrostatic Fields, 799
16.2 Use of Conformal Mapping, 804
16.3 Heat Problems, 808
16.4 Fluid Flow, 812
16.5 Poisson's Integral Formula, 819
16.6 General Properties of Harmonic Functions, 822

Chapter Review, 826
Chapter Summary, 827

Part E. Numerical Methods 828

Software 829

Chapter 17 Numerical Methods in General. 830
17.1 Introduction: Floating Point. Round-off, Error Propagation, etc., 831
17.2 Solution of Equations by Iteration, 838
17.3 Interpolation, 848
17.4 Splines, 861
17.5 Numerical Integration and Differentiation, 869
Chapter Review, 882
Chapter Summary, 884

Chapter 18 Numerical Methods in Linear Algebra 886
18.1 Linear Systems: Gauss Elimination, 886
18.2 Linear Systems: LU-Factorization, Matrix Inversion, 894
18.3 Linear Systems: Solution by Iteration, 900
18.4 Linear Systems: Ill-Conditioning, Norms, 906
18.5 Method of Least Squares, 914
18.6 Matrix Eigenvalue Problems: Introduction, 917
18.7 Inclusion of Matrix Eigenvalues, 920
18.8 Eigenvalues by Iteration (Power Method), 925
18.9 Tridiagonalization and QR-Factorization, 929
Chapter Review, 938
Chapter Summary, 940

Chapter 19 Numerical Methods for Differential Equations 942
19.1 Methods for First-Order Differential Equations, 942
19.2 Multistep Methods, 952
19.3 Methods for Systems and Higher Order Equations, 956
19.4 Methods for Elliptic Partial Differential Equations, 962
19.5 Neumann and Mixed Problems. Irregular Boundary, 971
19.6 Methods for Parabolic Equations, 976
19.7 Methods for Hyperbolic Equations, 982
Chapter Review, 984
Chapter Summary, 987

Part F. Optimization, Graphs 989

Chapter 20 Unconstrained Optimization, Linear Programming 990
20.1 Basic Concepts. Unconstrained Optimization, 990
20.2 Linear Programming, 994
20.3 Simplex Method, 998
20.4 Simplex Method: Degeneracy, Difficulties in Starting, 1002
Chapter Review, 1007
Chapter Summary, 1008

Chapter 21 Graphs and Combinatorial Optimization 1010
21.1 Graphs and Digraphs, 1010
21.2 Shortest Path Problems. Complexity, 1015
21.3 Bellman's Optimality Principle. Dijkstra's Algorithm, 1020
21.4 Shortest Spanning Trees. Kruskal's Greedy Algorithm, 1024