3.4. 2×2 - systeemien Kriittisten pisteiden luonne ominaisarvoja laskematta

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad D(\lambda) = \begin{vmatrix} a_{11}-\lambda & a_{12} \\ a_{21} & a_{22}-\lambda \end{vmatrix}$$

$$= \lambda^2 - \underbrace{(a_{11} + a_{22})}_{tr(A)}\lambda + \det A$$

$$= (\lambda - \lambda_1)(\lambda - \lambda_2) = \lambda^2 - (\lambda_1 + \lambda_2)\lambda + \lambda_1\lambda_2$$

$$\Rightarrow \begin{cases} \lambda_1 + \lambda_2 = tr(A) \\ \lambda_1\lambda_2 = \det(A) \end{cases}$$

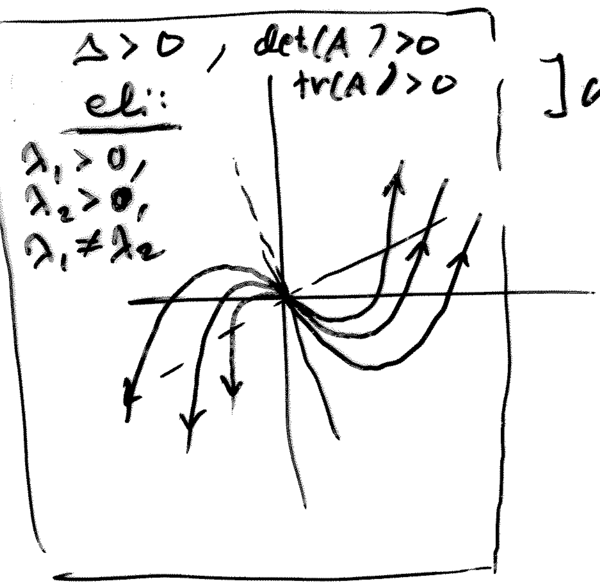
$$\Delta = tr(A)^2 - 4\det A \quad \text{(diskriminantti)}$$

I $\quad \Delta > 0, \quad \lambda_1, \lambda_2 \in \mathbb{R}, \ \lambda_1 \neq \lambda_2$

1. $\lambda_1\lambda_2 = \det(A) > 0 \Rightarrow$ noodi

Jos $tr(A) = \lambda_1 + \lambda_2 > 0$, lähde, epästabiili

Jos $tr(A) < 0$, nielu, vahvasti stabiili



$$\vec{y}(t) = c_1 e^{\lambda_1 t}\vec{x}_1 + c_2 e^{\lambda_2 t}\vec{x}_2$$

2. $\lambda_1\lambda_2 = \det A < 0 \Rightarrow$ satula epästab.