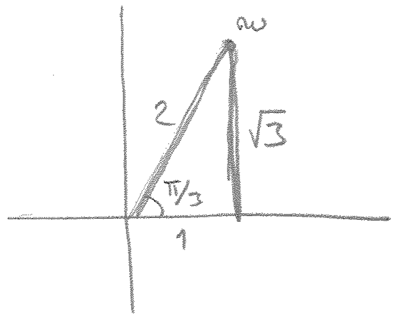
K3/P3 uusintavälikoe 1, 30.1.2006, HA

1) $z^6 = w = 1 + i\sqrt{3} \iff z = w^{1/6}$.



$|w| = 2$

$\varphi = \operatorname{Arg} w = \frac{\pi}{3}$

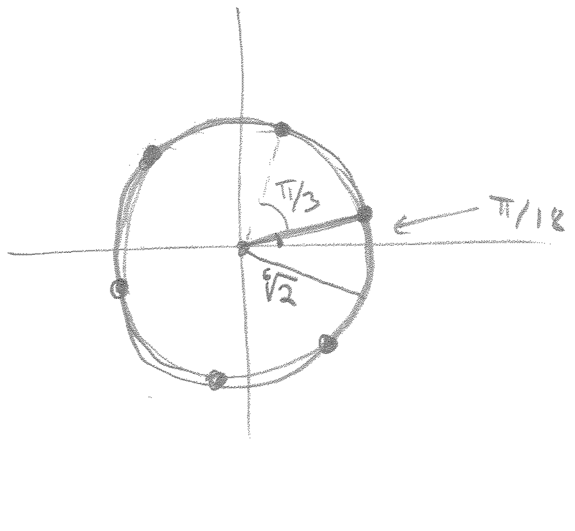
$z = \sqrt[6]{w} =$

$= \sqrt[6]{2} \cdot e^{i\left(\frac{\varphi}{6} + \frac{k \cdot 2\pi}{6}\right)}, \quad k = 0, 1, \dots, 5$

$= \sqrt[6]{2}\left(\cos\left(\frac{\pi}{18} + k \cdot \frac{\pi}{3}\right) + i \sin\left(\frac{\pi}{18} + k \cdot \frac{\pi}{3}\right)\right),$

$k = 0, 1, \dots, 5$

Likiarvot:
$1.105 + i\,0.195$
$0.384 + i\,1.055$
$-0.722 + i\,0.860$
$-1.105 - i\,0.195$
$-0.384 - i\,1.055$
$0.722 - i\,0.860$



2, 4) Kts. tentti.

3) (a) $f(t) = \sin t$, $f'(t) = \cos t$,
$f''(t) = -\sin t$

$$\mathcal{L}f'' = s^2 \mathcal{L}f - s\underbrace{f(0)}_{0} - \underbrace{f'(0)}_{1}$$
$$\| $$
$$-\mathcal{L}\{\sin t\} = -\mathcal{L}f \quad \Rightarrow (\mathcal{L}f)(s^2+1) = 1$$

Ratkaistaan: $\underline{\mathcal{L}f = \dfrac{1}{s^2+1}}$

(b) $$F(s) = \frac{3s+4}{\underbrace{s^2+4s+13}_{\underbrace{s^2+4s+4}_{(s+2)^2}+9}}$$

Täydennetään nimittäjässä neliöksi

$$\Rightarrow F(s) = \frac{3(s+2) - 3\cdot 2 + 4}{(s+2)^2+9}$$

$$= \frac{3(s+2) - 2}{(s+2)^2+9} = \left[\frac{3s}{s^2+9}\right]_{s\leftarrow s+2} - \frac{2}{3}\left[\frac{3}{s^2+9}\right]_{s\leftarrow s+2}$$

s-siirto $\Rightarrow$

$$\mathcal{L}^{-1}\{F(s)\} = 3e^{-2t}\cos 3t - \frac{2}{3}e^{-2t}\sin 3t$$