Computational Methods in Inverse Problems, Mat–1.3626

Fall 2007 Erkki Somersalo/Knarik Tunyan

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Consider the deconvolution problem discussed in the class. Instead of a first order ARMA model, write a second order model,

$$X_j = \frac{1}{2}(X_{j-1} + X_{j+1}) + \sqrt{\theta_j} W_j,$$

assuming the boundary conditions

$$X_0 = X_{n+1} = 0.$$

1. Write the ARMA model in matrix form, and find the corresponding prior model, first assuming stationarity, $\theta_j = \theta_0 =$ constant.
2. Modify the first order Matlab code so that it solves the deconvolution problem with the stationary assumption. Try different values of θ_0 to get an idea how the algorithm performs with different input signals (boxcar, triangular pulse, smooth oscillation as in the first order model program `deconvolution.m`).
3. Modify now the model by letting θ_j's be different but known. Write a matrix form of the ARMA model and the corresponding prior density.
4. Assume that $\theta_j = \theta_0$=constant except for few values of j, where the value θ_j is higher, e.g., $\theta_j = 100\theta_0$. Plot a few random draws from the corresponding prior.
5. Consider the triangular input function. Assume that you know a priori where the maximum of the signal is. Try the deconvolution with your program, setting the variance θ_j at the maximum higher than elsewhere.