Mat-1.3379 Dynaamisten systeemien erikoiskurssi **Eirola/Hannukainen**
Harjoitus 6, 2.3.2006 12-14 Y338C

Harjoitus 6 on tietokoneharjoitus. Tehtäviä tehdään yhdessä assistentin kanssa tietokoneluokassa ja joistain tehtävistä palautetaan lyhyt selostus 16.3. harjoituksiin mennesä.

1. (Problem 5.6. 135)
   Consider the *logistic map*

$$x^+ = \alpha x(1-x)\,, \quad \alpha > 1\,.$$

   Find the value α_0 where the map undergoes a flip bifurcation, and check the nondegeneracy conditions there. Next, find analytically the value α_1 where the second iterate undergoes period doubling, and check nondegeneracy there. Then, find α_k, the values where the 2^k–th iterate undergoes period doubling; do this at least up to $k = 6$ (the more, the better). For $k = 2$ you should be able to find α_k analytically, then you ought to device a suitable approximation method to find the α_k's. For $k \geq 2$, you may avoid checking the nondegeneracy conditions. Compute the ratios

$$\frac{\alpha_k - \alpha_{k-1}}{\alpha_{k+1} - \alpha_k}\,,$$

   and verify that they are approaching the value $\gamma_F = 4.6692\ldots$. This is a universal constant known as Feigenbaum's number. The period doubling phenomenon you just observed for the logistic map, and the limiting behavior of the ratios, are typical of many other maps.

2. (Problem 5.11. s.136) Consider the planar map

$$\begin{bmatrix} x^+ \\ y^+ \end{bmatrix} = \boldsymbol{\phi}(x,y) = \begin{bmatrix} y \\ \alpha - x^2 \end{bmatrix}.$$

   Show that the fixed point $x = y = -1/2 + \sqrt{1/4 + \alpha}$ changes from stable to unstable as α crosses $3/4$, where the eigenvalues of $D\boldsymbol{\phi}$ cross the unit circle at $\pm i$. Thus, there is a resonance in the Neimark–Sacker bifurcation. Show, analytically or by numerical simulation, that as α increases through $3/4$ there are two orbits of period 4 whose iterates lie on the vertices of a square and somewhere along the square's sides (they are close to the midpoints of these sides, for α close to $3/4$). The square is invariant under the map, it is a non-smooth invariant circle.

3. Consider the boundary value problem

$$\begin{cases} u' &= v, u(0) = 0, u(c) = b \\ v' &= \mu\sqrt{1+v^2} \\ \mu' &= 0, \\ s' &= \sqrt{1+v^2}. \end{cases}$$

   Fix a=1, b=4, c=4, L=10 and solve the problem by using the implicite midpoint rule for time discretization and Newton's method to solve the nonlinear system. Use $u(t) = a + (b-a)t/c + \frac{L-c}{2c}t(t-c)$ as an initial guess.

   Palauta (16.3. Mennessä) kuva tehtävän ratkaisusta. Kurssihakemistossa on puolivalmis ohjelma tehtävän ratkaisemiseen. Lue ja täydennä bvp_Dfu.m ja bvp_fu.m - funktiot. (Tarvitset näitä myös seuraavan tehtävän ratkaisuun.)

4. (Problem 3.6 p. 219) Consider the Lorentz system:

$$\begin{cases} x' &= -\sigma x + \sigma y \\ y' &= \alpha x - xz - y \\ z' &= xy - bz\,. \end{cases}$$

Fix $\sigma = 10$, $b = 8/3$, and let α be the free parameter. First, compute the curve of stationary solutions for $\alpha \in [0, 30]$, starting from the trivial solution $(0, 0, 0)$. There should be a pitchfork bifurcation at $\alpha = 1$, and then again there will be a Hopf point along both bifurcating branches at $\alpha = \alpha_H \approx 24.73684$, at which point the branches lose their stability. (Because of symmetry, these two branches look identical, just reversed). Compute the branch of periodic orbits emanating at the Hopf point (notice that you will have periodic orbits for $\alpha < \alpha_H$).

Palauta (16.3. Mennessä) kuva, jonka saat aikaan ajamalla annettuja tiedostoja (Täydennä sopivilta osin tht4.m - tiedostoa) , selostus siitä, kuinka sait seurattua kaikkia haaroja ja periodisen ratkaisun pituus α:n funktiona.