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PREFACE

This book is a text for a first-year graduate course in complex analysis. All material usually treated in such a course is covered here, but our book is based on principles that differ somewhat from those underlying most introductory graduate texts on the subject.

First of all, we have developed the idea that an introductory book on this subject should emphasize how complex analysis is a natural outgrowth of multivariable real calculus. Complex function theory has, of course, long been an independently flourishing field. But the easiest path into the subject is to observe how at least its rudiments arise directly from ideas about calculus with which the student will already be familiar. We pursue this point of view both by comparing and by contrasting complex variable theory with real variable calculus.

Second, we have made a systematic attempt to separate analytical ideas, belonging to complex analysis in the strictest sense, from topological considerations. Historically, complex analysis and topology grew up together in the late nineteenth century. And, long ago, it was natural to write complex analysis texts that were a simultaneous introduction to both subjects. But topology has been an independent discipline for almost a century, and it seems to us only a confusion of issues to treat complex analysis as a justification for an introduction to the topology of the plane. Topological questions do arise naturally, of course; but we have collected all of the difficult topological issues in a single chapter (Chapter 11), leaving the way open for a more direct and less ambivalent approach to the analytical material.

Finally, we have included a number of special topics in the later chapters that bring the reader close to subjects of current research. These include the Bergman kernel function, H^p spaces, and the Bell-Ligočka approach to proving smoothness to the boundary of biholomorphic mappings. These topics are not part of a standard course on complex analysis (i.e., they would probably not appear on any qualifying examination), but they are in fact quite accessible once the standard material is mastered.

A large number of exercises are included, many of them being routine drill but many others being further developments of the theory that the reader can carry