Mat-1.C Matemaattiset ohjelmistot

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[www.math.hut.fi/teaching/MatOhjelmistot/maple/harj1ratk.mw]
/harj1ratk.pdf

Aloitetaan Maple-opiskelu vertailemalla harj1:n Maple- ja Matlab-ratkaisuja niiltä osin kuin mielekästä.

Tehtävät: 2,3,4,5,6,13,14,15,16.

▼ 2.

```
> with(LinearAlgebra) :
> alias(Tr = Transpose);
```

$$Tr \tag{1.1}$$

```
> x := Tr(⟨ x1, x2, x3, x4, x5⟩);
```

$$x := \begin{bmatrix} x1 & x2 & x3 & x4 & x5 \end{bmatrix} \tag{1.2}$$

```
> y := ⟨y1 | y2 | y3 |y4 ⟩;
```

$$y := \begin{bmatrix} y1 & y2 & y3 & y4 \end{bmatrix} \tag{1.3}$$

```
> z := ⟨z1|z2 |z3 |z4 |z5 ⟩;
```

$$z := \begin{bmatrix} z1 & z2 & z3 & z4 & z5 \end{bmatrix} \tag{1.4}$$

```
> x·z;
Error, (in rtable/Product) invalid arguments
> x.z;
```

$$\overline{z1}\, x1 + \overline{z2}\, x2 + \overline{z3}\, x3 + \overline{z4}\, x4 + \overline{z5}\, x5 \tag{1.5}$$

```
> x.Tr(z);
```

$$z1\, x1 + z2\, x2 + z3\, x3 + z4\, x4 + z5\, x5 \tag{1.6}$$

```
> x^2;
Error, (in rtable/Power) exponentiation operation not defined
for Vectors
> map(t→t^2, x);
```

$$\begin{bmatrix} x1^2 & x2^2 & x3^2 & x4^2 & x5^2 \end{bmatrix} \tag{1.7}$$

```
> Tr(x).y;  # Ulkotulo
```

$$\begin{bmatrix} x1\, y1 & x1\, y2 & x1\, y3 & x1\, y4 \\ x2\, y1 & x2\, y2 & x2\, y3 & x2\, y4 \\ x3\, y1 & x3\, y2 & x3\, y3 & x3\, y4 \\ x4\, y1 & x4\, y2 & x4\, y3 & x4\, y4 \\ x5\, y1 & x5\, y2 & x5\, y3 & x5\, y4 \end{bmatrix} \tag{1.8}$$

```
> s2 := sum(x[i], i = 1 .. Dimension(x));  # Aika outo virhe. Mutta klikkaa Erroria, mahtavaa!
```

> $s2 := \text{sqrt}\left(add\left((x[i])^2, i = 1 .. Dimension(x)\right)\right);$

$$s2 := \sqrt{x1^2 + x2^2 + x3^2 + x4^2 + x5^2} \tag{1.9}$$

> $s3 := norm(x, 2);$

$$s3 := \sqrt{|x1|^2 + |x2|^2 + |x3|^2 + |x4|^2 + |x5|^2} \tag{1.10}$$

> $Dimension(x);$

$$5 \tag{1.11}$$

> $x[3];$

$$x3 \tag{1.12}$$

> $i;$

$$i \tag{1.13}$$

▼ 3.

Maple	Matlab
> $C := \langle\langle 1, 1, 1 \rangle | \langle 2, 3, 4 \rangle | \langle 3, 6, 9 \rangle\rangle;$ $C := \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 6 \\ 1 & 4 & 9 \end{bmatrix}$ (2.1) >	> C=[1 2 3; 1 3 6; 1 4 9]; tai C=[ones(3,1) (2:4)' (3:3:9)']
> $C^2;$ $\begin{bmatrix} 6 & 20 & 42 \\ 10 & 35 & 75 \\ 14 & 50 & 108 \end{bmatrix}$ (2.2) > $map(x \rightarrow x^2, C);$ $\begin{bmatrix} 1 & 4 & 9 \\ 1 & 9 & 36 \\ 1 & 16 & 81 \end{bmatrix}$ (2.3) >	> C^2 C.^2

▼ 4.

> $restart;$

                             >> clear

> $y := 5 \cdot \cos(3 \cdot Pi \cdot x);$

$$y := 5 \cos(3\pi x)$$ **(3.1)**

> $plot(y, x = -5 .. 5);$

Tai funktiom"a"aritys:

> $f := x \rightarrow 5 \cdot \cos(3 \cdot Pi \cdot x);$

$$f := x \rightarrow 5 \cos(3\pi x)$$ **(3.2)**

> $plot(f(x), x = -5 .. 5);$

>

c)

> $y := \dfrac{\sin(7 \cdot x) - \sin(5 \cdot x)}{\cos(7 \cdot x) + \cos(5 \cdot x)};$

$$y := \frac{\sin(7x) - \sin(5x)}{\cos(7x) + \cos(5x)}$$ **(3.3)**
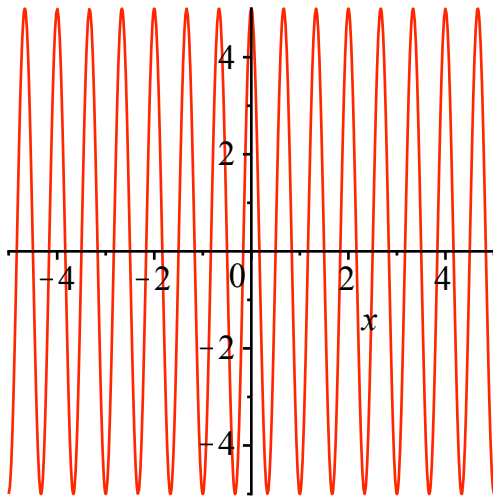
> $plot(denom(y), x = -2 .. 2);$
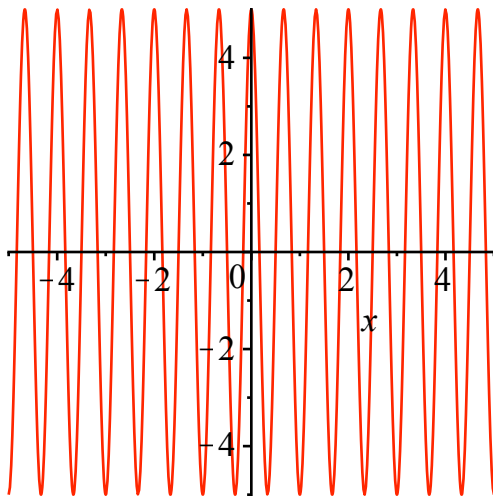
```
x=linspace(-5,5); y=5*cos(3*pi*x);plot(x,y)
>> f=@(x) 5*cos(3*pi*x)
f =
    @(x)5*cos(3*pi*x)
>> fplot(f,[-5,5])
>> shg
```
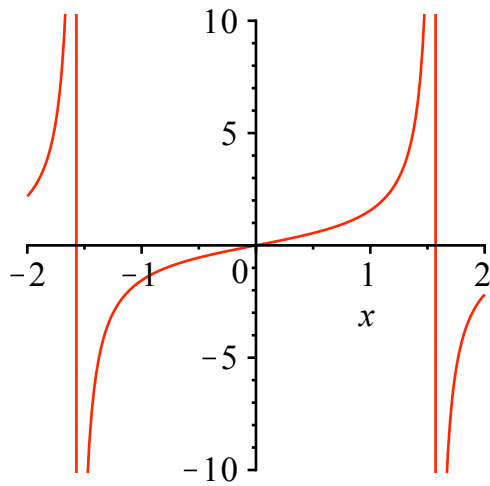
```
x=linspace(-2,2);
oso=sin(7*x)-sin(5*x);
nimi=cos(7*x)+cos(5*x);
plot(x,nimi);shg
x=linspace(-2,2,1000);
oso=sin(7*x)-sin(5*x);
nimi=cos(7*x)+cos(5*x);
```
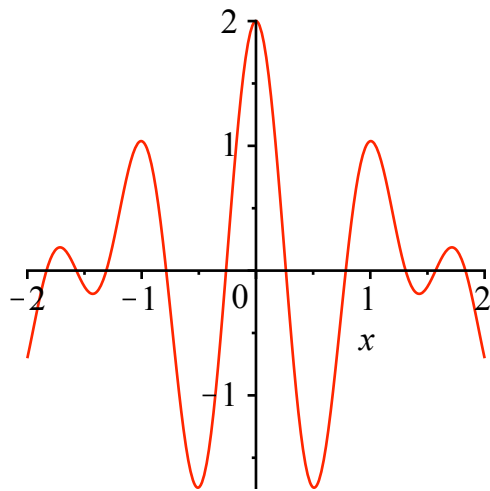
```
plot(x,oso./nimi);shg
ylim([-10 10]);grid on; shg
```

> $plot(y, x = -2..2, -10..10);$

>

5.

Maple h"avi"a"a t"ass"a **Matlabi**lle, onnistuuhan nuo, mutta hankalammin ja tehottomammin.

Valitaan nyt se ty"okalu, Matlab, joka t"ah"an sopii.

a)
```
>> format compact
>> A=1:10
A =
    1   2   3   4   5   6   7   8   9   10
>> A>5
ans =
    0   0   0   0   0   1   1   1   1   1
>> sum(ans)
ans =
    5
>> find(A>5)
```

b)
```
>> format compact
>> B=[1 2 3;4 5 6;7 8 9]
B =
    1   2   3
    4   5   6
    7   8   9
>> sum(B>5)
ans =
    1   1   2
>> B>5
ans =
```

```
ans =
    6    7    8    9   10
>> length(ans)
ans =
    5
```

Yleisp"atevat (vekt. ja matr.)

```
>> sum(sum(Data>5))
>> length(find(Data>5))
```

```
    0    0    0
    0    0    1
    1    1    1
>> sum(B>5)
ans =
    1    1    2
>> sum(sum(B>5))
ans =
    4
>> find(B>5)
ans =
    3
    6
    8
    9
>> length(find(B>5))
ans =
    4
>>


>> A=1:10
A =
    1    2    3    4    5    6    7    8    9   10
>> A>5
ans =
    0    0    0    0    0    1    1    1    1    1
>> sum(ans)
ans =
    5
>> find(A>5)
ans =
    6    7    8    9   10
>> length(ans)
ans =
    5
```

6.

```
> restart;
>
```

```
A=[4 -5;2 1]
```

```
> with(LinearAlgebra) :
> alias(Lsolve = LinearSolve);
                        Lsolve                              (5.1)
```
$$A := \langle\, \langle\, 4, 2\rangle \mid \langle -5, 1\rangle\, \rangle;$$
$$A := \begin{bmatrix} 4 & -5 \\ 2 & 1 \end{bmatrix} \qquad (5.2)$$
$$b := \langle\, 11, 9\,\rangle;$$
$$b := \begin{bmatrix} 11 \\ 9 \end{bmatrix} \qquad (5.3)$$
$$xy := Lsolve(A, b);$$
$$xy := \begin{bmatrix} 4 \\ 1 \end{bmatrix} \qquad (5.4)$$
```
>
```
Viel"a helpommin suoraan yleisellä solve:lla:

```
> restart;
> yhtalot := {4·x − 5·y = 11, 2·x + y
        = 9};
```
$$yhtalot := \{2\,x + y = 9,\ 4\,x - 5\,y = 11\} \qquad (5.5)$$
```
> ratk := solve(yhtalot, {x, y});
          ratk := {x = 4, y = 1}                            (5.6)
> x;
                        x                                   (5.7)
> X := subs(ratk, x);
              X := 4                                        (5.8)
> Y := subs(ratk, y);
              Y := 1                                        (5.9)
> subs(ratk, yhtalot);
          {9 = 9, 11 = 11}                                  (5.10)
```
  Helpompi, mutta "vaarallisempi":
```
> assign(ratk);
> x;
                        4                                   (5.11)
> y;
                        1                                   (5.12)
> subs(ratk, yhtalot);
          {9 = 9, 11 = 11}                                  (5.13)
```
Kas, odottelin virhett"a, mutta ei tullutkaan, voi harmi!
```
>
```

```
> restart;
```

```
b=[11;9]
xy=A\b
>> A=[4 -5;2 1]
b=[11;9]
xy=A\b
A =
    4   -5
    2    1
b =
   11
    9
xy =
    4
    1
```

```
C1 = F1*a +b
```

> $kaava := C = a \cdot F + b;$

$$kaava := C = a\,F + b \qquad \text{(5.14)}$$

> $yht1 := subs(C = 0, F = 32, kaava);$

$$yht1 := 0 = 32\,a + b \qquad \text{(5.15)}$$

> $yht2 := subs(C = -40, F = -40, kaava);$

$$yht2 := -40 = -40\,a + b \qquad \text{(5.16)}$$

> $ratk := solve(\{yht1, yht2\}, \{a, b\});$

$$ratk := \left\{ a = \frac{5}{9},\, b = -\frac{160}{9} \right\} \qquad \text{(5.17)}$$

>

> $solve(kaava, F);$

$$\frac{-b + C}{a} \qquad \text{(5.18)}$$

> $Ffun := C \to \dfrac{-b + C}{a};$

$$Ffun := C \to \frac{-b + C}{a} \qquad \text{(5.19)}$$

> $assign(ratk);$

> $Ffun(C);$

$$32 + \frac{9}{5}\,C \qquad \text{(5.20)}$$

>

> $h := 5;$

$$h := 5 \qquad \text{(5.21)}$$

> $pisteet := seq([-50 + k \cdot h, Ffun(-50 + k \cdot h)], k = 0..10);$

$$\begin{aligned} pisteet := {}& [-50, -58], [-45, -49], [\\ & -40, -40], [-35, -31], [-30,\\ & -22], [-25, -13], [-20, -4], [\\ & -15, 5], [-10, 14], [-5, 23], [0,\\ & 32] \end{aligned} \qquad \text{(5.22)}$$

> $matrix([pisteet]);$

$$\qquad \text{(5.23)}$$

```
C2 = F2*a+b
A=[F1  1;
   F2  1]
B=[C1;C2]
Nyt Matlabiin:
>>>>>>>>>>>>>>>>>>>>>>>>>>
F1=32;C1=0;
F2=-40;C2=-40;
A=[F1  1;
   F2  1]
B=[C1;C2]
format rat
ab=A\B
a=ab(1);b=ab(2);
<<<<<<<<<<<<<<<<<<<<<<<<<<<
```

$$F = \frac{C - b}{a}$$

```
>>>>>>>>>>>>>>>>>>>>>>>>>>>
C=(-50:5:20)';
F=(C-b)/a;
[C F]
```

Matlab-ajo komentoikkunassa:

```
>> F1=32;C1=0;
F2=-40;C2=-40;
A=[F1  1;
   F2  1]
B=[C1;C2]
format rat
ab=A\B
a=ab(1);b=ab(2);
A =
     32          1
    -40          1
B =
      0
    -40
ab =
     5/9
   -160/9
>> C=(-50:5:20)';
F=(C-b)/a;
[C F]
ans =
    -50          -58
    -45          -49
    -40          -40
    -35          -31
```

$$\begin{bmatrix} -50 & -58 \\ -45 & -49 \\ -40 & -40 \\ -35 & -31 \\ -30 & -22 \\ -25 & -13 \\ -20 & -4 \\ -15 & 5 \\ -10 & 14 \\ -5 & 23 \\ 0 & 32 \end{bmatrix}$$ **(5.23)**

> $plot([pisteet])$;

-30	-22
-25	-13
-20	-4
-15	5
-10	14
-5	23
0	32
5	41
10	50
15	59
20	68

Voit kokeilla my"os

> # Matrix(pisteet);

Hiiren oikealla -> browse. N"atti taulukko,
jonka saa exportilla suoraan Excel:ksi.

>

13.

K"asitellaan lausekkeena:

> $f := x^4 - 7 \cdot x^3 + 11 \cdot x^2 + 7 \cdot x - 10$;

$f := x^4 - 7x^3 + 11x^2 + 7x - 10$ **(6.1)**

>

```
> plot(f, x = -2 .. 5, y = -15 .. 20);
```
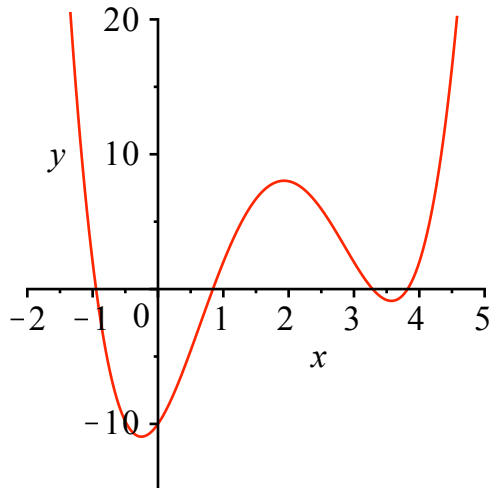


```
> factor(f);
```
$$x^4 - 7x^3 + 11x^2 + 7x - 10 \qquad \text{(6.2)}$$

```
> juuret := fsolve(f = 0, x);
```
$$juuret := -0.9474126425,$$
$$0.8407256773, 3.285864689,$$
$$3.820822276 \qquad \text{(6.3)}$$

```
> subs(x = juuret[1], f);
```
$$0. \qquad \text{(6.4)}$$

```
> seq(subs(x = juuret[i], f), i = 1 .. 4);
```
$$0., 0., -8. \ 10^{-8}, 3. \ 10^{-8} \qquad \text{(6.5)}$$

```
> Digits;
```
$$10 \qquad \text{(6.6)}$$

```
> Digits := 20;
```
$$Digits := 20 \qquad \text{(6.7)}$$

```
> Digits := 10;
```
$$Digits := 10 \qquad \text{(6.8)}$$

```
> df := diff(f, x);
```
$$df := 4x^3 - 21x^2 + 22x + 7 \qquad \text{(6.9)}$$

```
> plot(df, x = -2 .. 5);
```
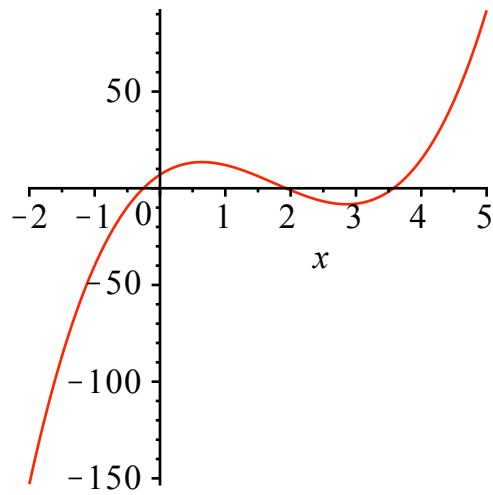
> $dnollak := fsolve(df = 0, x);$

$dnollak := -0.2537492459,$ **(6.10)**
$\quad 1.929535447, 3.574213799$

> $subs(x = dnollak[1], f);$

$\quad -10.94945329$ **(6.11)**

> $subs(x = dnollak[2], f);$

$\quad 8.03538256$ **(6.12)**

> $subs(x = dnollak[3], f);$

$\quad -0.87889801$ **(6.13)**

> $plot([f, df], x = -2 .. 4);$

>

14.
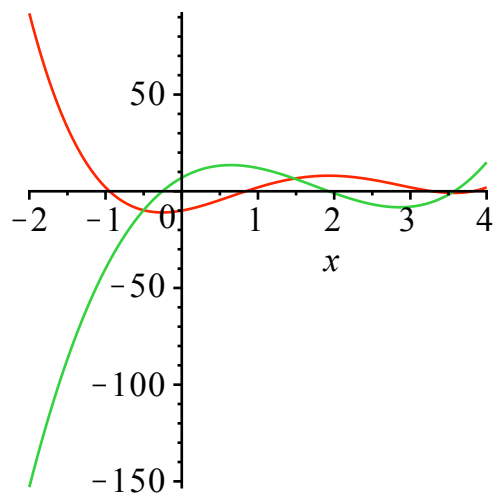
> $restart:$

> $plot([\cos(x), \sin(x)], x = -2 \cdot Pi .. 2 \cdot Pi,$

```
x=linspace(-2*pi,2*pi);
plot(x,cos(x),x,sin(x))
```
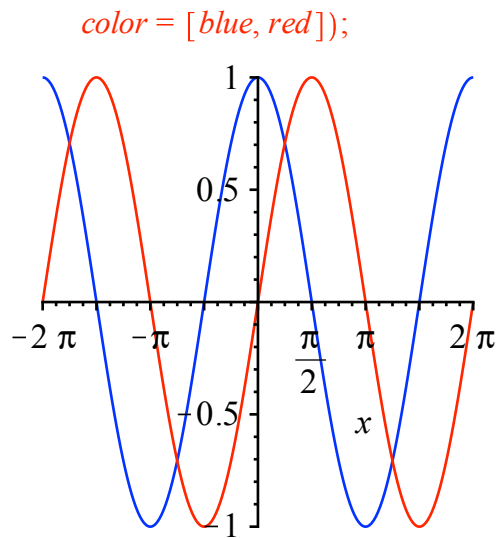
```
t=linspace(0,2*pi);
x=cos(t);
y=sin(t);
plot(x,y)
hold on
t=linspace(0,2*pi,10);
plot(cos(t),sin(t))
axis square
shg
```

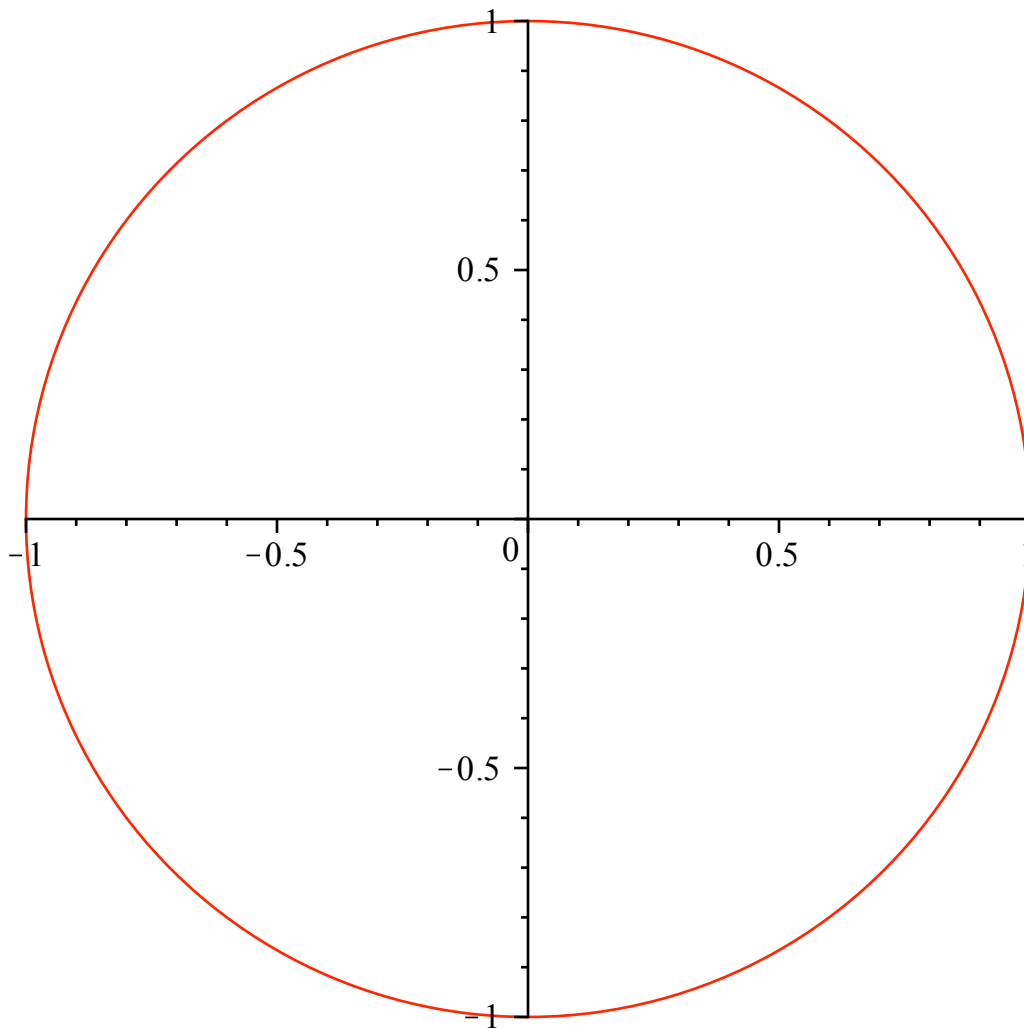> $plot([\cos(x), \sin(x)], x = -2 \cdot Pi .. 2 \cdot Pi, color = [blue, red]);$
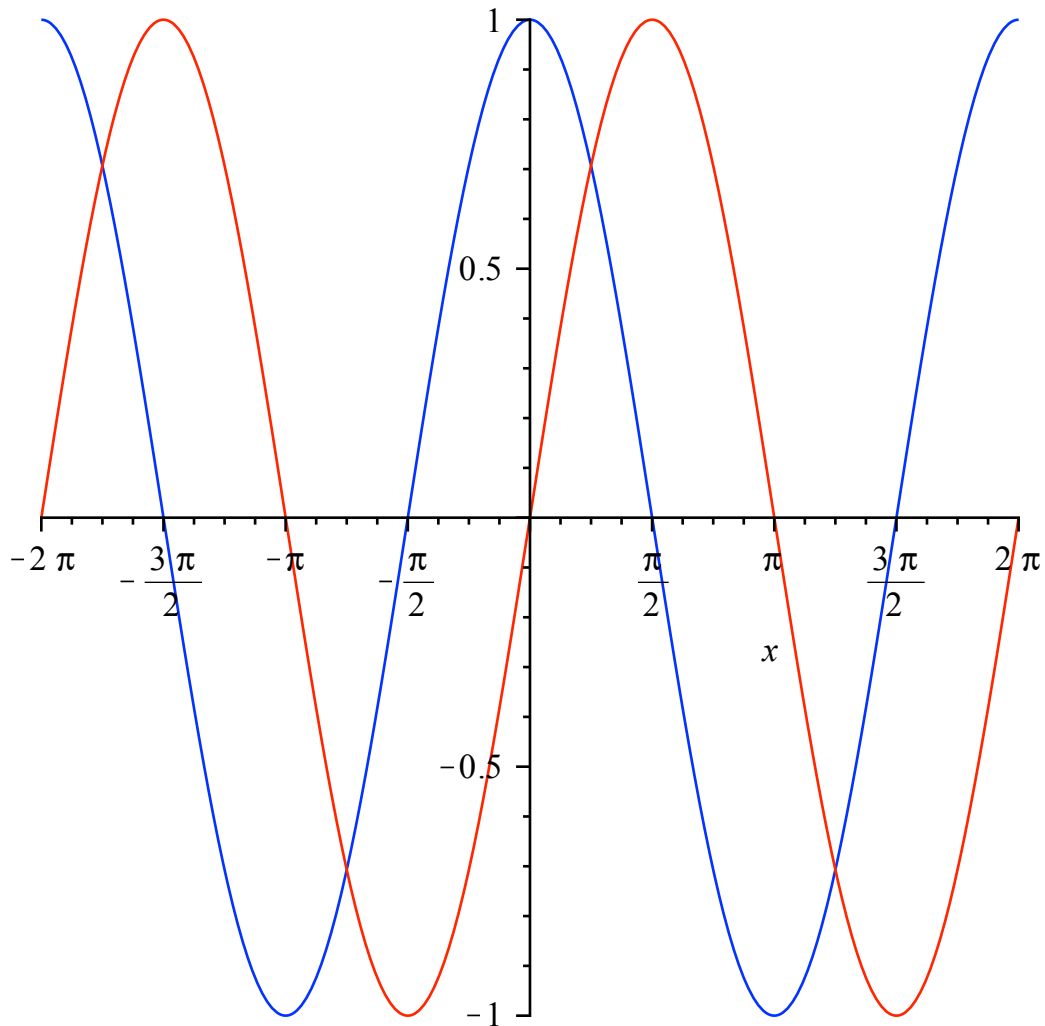
> $plot([\cos(t), \sin(t), t = -\mathrm{Pi} .. \mathrm{Pi}]);$
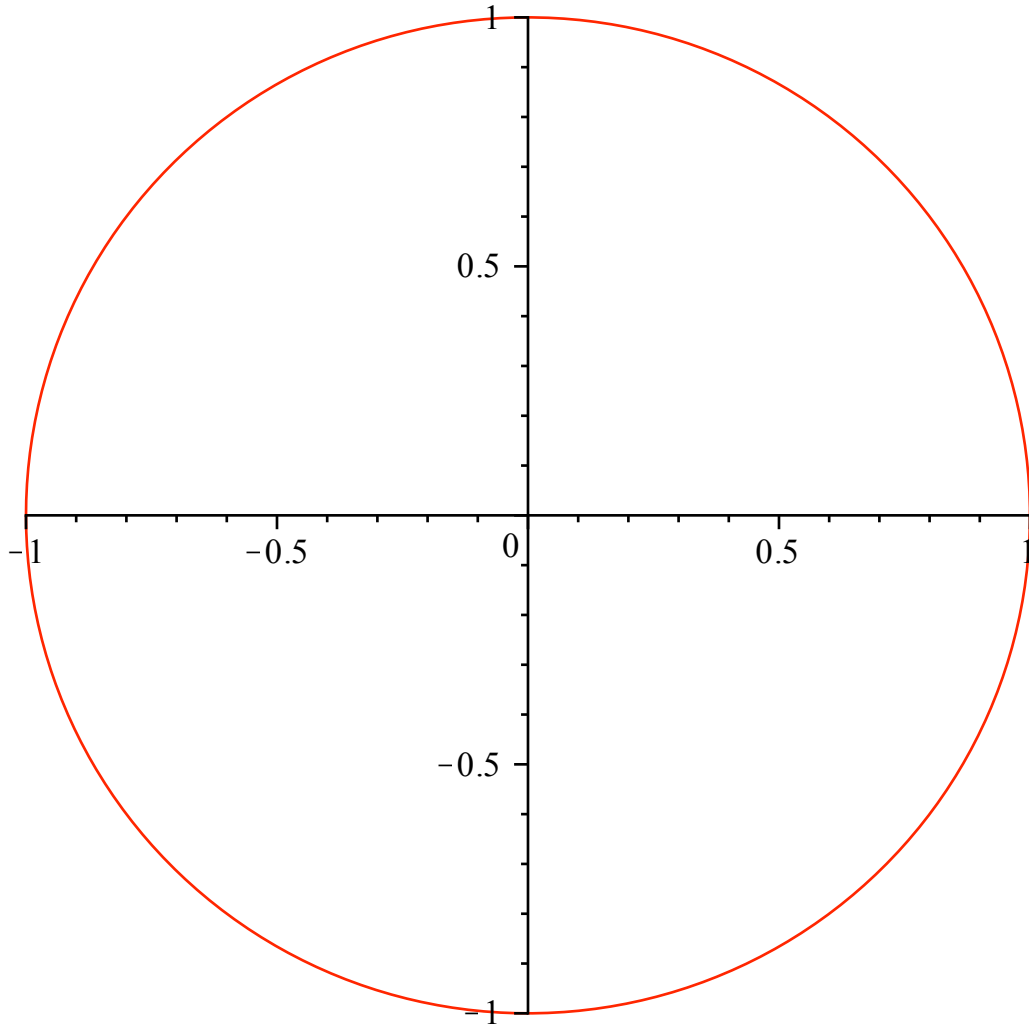
> $with(plots) :$

> $complexplot(\exp(I \cdot t), t = 0 .. 2 \cdot \mathrm{Pi});$

Oma linspace-funktio Mapleen:

Kun tähän on Matlabissa totuttu, niin määritellään Mapleen. Käytössä on Maplen seq, jolla muodostetaan jonoja. Kas näin se käy:

> $linspace := (a, b, n) \rightarrow [\,seq(a + iii * (b - a) / (n - 1), iii = 0 .. n - 1\,)\,];$

$$linspace := (a, b, n) \rightarrow \left[seq\left(a + \frac{iii\,(b - a)}{n - 1}, iii = 0 .. n - 1 \right) \right] \tag{7.1}$$

Yhden rivin määrittelyssä ei voi sisäisiä muuttujia lokalisoida, siksi i:n sijasta iii.
(Taiteen sääntöjen mukainen pitää kirjoittaa proc:ksi, laitetaan ohjelmekokoelmaamme.)

> $linspace(0, 1, 5);$

$$\left[0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1 \right] \tag{7.2}$$

> $t := linspace(0, 2 \cdot Pi, 11);$

$$t := \left[0, \frac{1}{5}\,\pi, \frac{2}{5}\,\pi, \frac{3}{5}\,\pi, \frac{4}{5}\,\pi, \pi, \frac{6}{5}\,\pi, \frac{7}{5}\,\pi, \frac{8}{5}\,\pi, \frac{9}{5}\,\pi, 2\,\pi \right] \tag{7.3}$$

> $x := \cos{\sim}(t); y := \sin{\sim}(t);$ # *Mato mappaa funktion, sama kuin map(cos,t)*

$x := \left[1, \cos\left(\frac{1}{5}\,\pi \right), \cos\left(\frac{2}{5}\,\pi \right), -\cos\left(\frac{2}{5}\,\pi \right), -\cos\left(\frac{1}{5}\,\pi \right), -1, -\cos\left(\frac{1}{5}\,\pi \right),$

$$-\cos\left(\frac{2}{5}\pi\right), \cos\left(\frac{2}{5}\pi\right), \cos\left(\frac{1}{5}\pi\right), 1\Bigg]$$

$$y := \Bigg[0, \sin\left(\frac{1}{5}\pi\right), \sin\left(\frac{2}{5}\pi\right), \sin\left(\frac{2}{5}\pi\right), \sin\left(\frac{1}{5}\pi\right), 0, -\sin\left(\frac{1}{5}\pi\right), -\sin\left(\frac{2}{5}\pi\right),$$ (7.4)

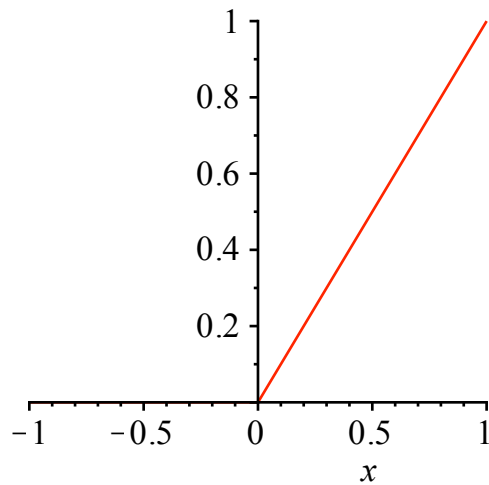$$-\sin\left(\frac{2}{5}\pi\right), -\sin\left(\frac{1}{5}\pi\right), 0\Bigg]$$

> $plot(x, y);$
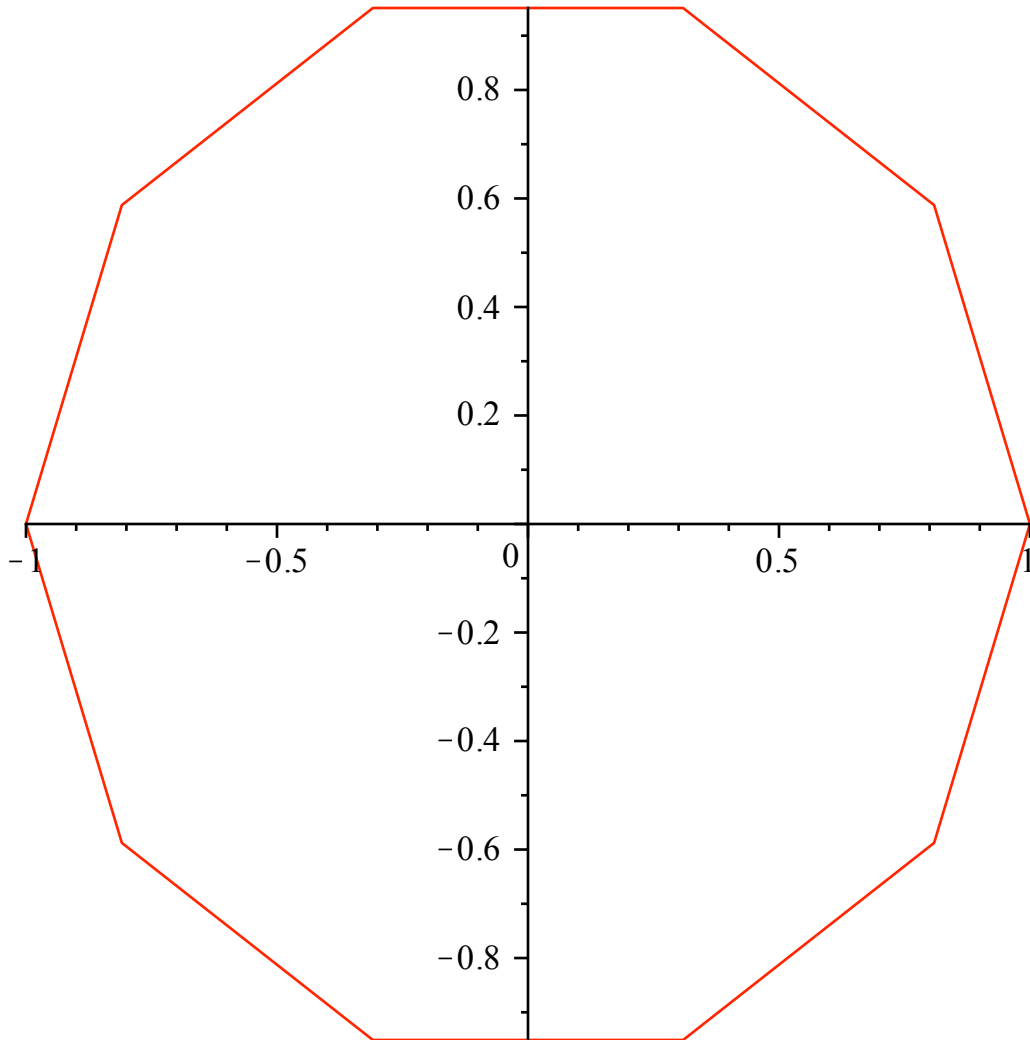
15.

> $restart;$

> $g := x \rightarrow piecewise(x > 0, x, x < 0, 0);$
 $g := x \rightarrow piecewise(0 < x, x, x < 0, 0)$ **(8.1)**
 > $plot(g(x), x = -1..1);$
	x=linspace(-2,2,10);
	 nolla=zeros(size(x));
	 y=max(x,nolla);
	
	 tai:

```
z=(x>0).*x

Tiedostoon ramppi1.m kirjoitetaan:
-------------------------------------------------
function [ y ] = ramppi1( x )
% Lasketaan "ramppifuktion" arvot x-vektorissa.
y=max(zeros(size(x)),x);
end
-------------------------------------
help ramppi1
Kutsuesim:
x=linspace(-2,2);
plot(x,ramppi1(x));shg
```

▼ 16.

> *restart*;

> $plot\left([\cos(x), x], x = 0 .. \dfrac{Pi}{2} \right)$;
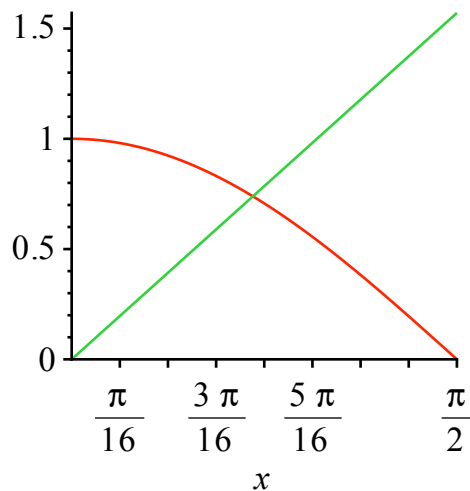

```
x=linspace(0,pi/2);
plot(x,cos(x),'r',x,x,'g')
>> grid on;shg
>> x0=0.739
x0 =
    0.7390
>> cos(x0)
ans =
    0.7391
```

> *x0* := 0.737;
$$x0 := 0.737 \qquad (9.1)$$

> $\cos(x0)$;
$$0.7404880963 \qquad (9.2)$$

> *x0* := $\cos(x0)$;
$$x0 := 0.7404880963 \qquad (9.3)$$

>

>