# Conference on Numerical Matrix Analysis and Operator Theory

September 3-5, 2008

Helsinki University of Technology
Institute of Mathematics

as a part of

**Special Year in Numerics 2008-2009 organized jointly with IMPAN**

## Special Year in Numerics

The Finnish Mathematical Society has chosen Numerical Analysis as the theme for its visitor program for the period 2008-2009. The main events of Special Year in Numerics are meetings and short courses organized between May 2008 and June 2009. These are planned to take place in connection with the 100th anniversary celebration of Helsinki University of Technology in 2008.

## Aim of the conference

The purpose of the conference is to bring together people from a wide range of numerical matrix analysis and operator theory, to learn about mathematics from the perspective of multiple fields, and to meet a diverse group of people and have an opportunity to form new collaborations.

## Scientific committee

Marko Huhtanen
Olavi Nevanlinna
Yuriy Tomilov
Jaroslav Zemánek

## Acknowledgment

Special Year in Numerics is sponsored by the Academy of Finland, the European Science Foundation through the European Scientific Network *Advanced Mathematical Methods for Finance* (AMaMeF), as well as the Väisälä Foundation.

# 1 General information

## Directions

Helsinki University of Technology (HUT) is located in Otaniemi, Espoo. It is a five minutes walk from Hotel Radisson SAS Espoo to the main building of HUT.

The talks will be held in the lecture hall E which is located on the ground floor of the main building of HUT. The registration desk is located outside the lecture hall E.

## Lunches

There is the restaurant *Alvari* on the ground floor of the main building near the lecture hall E. The opening hours are 8.00 - 18.00 and lunch is served 10.45 - 13.30.

There is also the restaurant *Dipoli* which is located in the campus. Lunch is served 10.30 - 16.00.

## Computer access

Instructions about computers and wlan with passwords will be handed out at the registration desk. Please handle the password sheet responsibly.

## Social events

On Thursday at 18.00 the conference dinner will be held at the Saha hall. The Saha hall is located about 250 meters from the main building of Helsinki University of Technology.

## Tourist information and activities in Helsinki

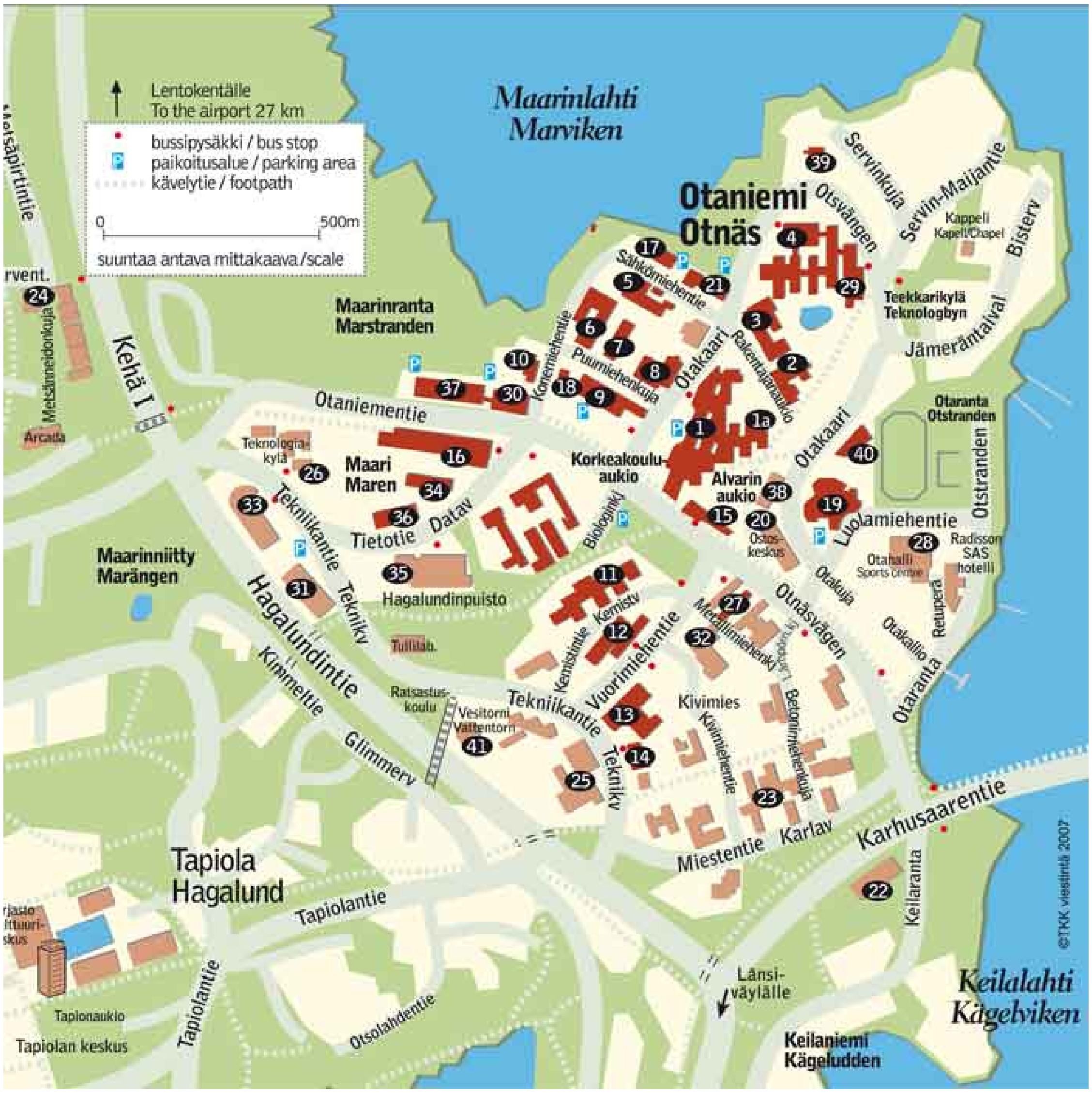
The city of Helsinki offers a lot to see and experience for visitors. The heart of Helsinki consists of Senate Square and Market Square. The National Museum of Finland, the Ateneum Art Museum as well as the Museum of Contemporary Art Kiasma are all within five minutes walking distance from there. Some of the other most popular sights in Helsinki include Suomenlinna Maritime fortress, Linnanmäki Amusement Park and Korkeasaari Zoo.

More information about activities in Helsinki can be found at

http://www.hel2.fi/tourism/en/matko.asp

The buses 102 and 103 commute between Otaniemi and downtown Helsinki.

## 2 Map of Otaniemi



# 3 Program

**Wednesday**

| | |
|---|---|
| 9:15 | Registration and coffee |
| 9:45 | Opening remarks |
| | *Chair:* Seppo Hassi |
| 10:00 | Charles Batty: Rates of decay of smooth orbits of semigroups of operators |
| 10:30 | Jani Virtanen: Norms of Toeplitz and Hankel matrices and their asymptotic behavior |
| 11:00 | Iwona Wróbel: On the Gauss-Lucas theorem, the numerical range and the Sendov conjecture |
| 11:30 | David Shoikhet: Old and new in complex dynamics |
| 12:15 | Lunch |
| | *Chair:* Anne Greenbaum |
| 13:30 | Thomas Laffey: The influence of appending zeros to the spectrum in the nonnegative inverse eigenvalue problem |
| 14:00 | Mikael Lindström: Essential norm of operators on weighted Banach spaces of analytic functions |
| 14:30 | Krystyna Zietak: On some known and open matrix nearness problems |
| 15:00 | Coffee |
| | *Chair:* Charles Batty |
| 15:30 | Seppo Hassi: Characterization of eigenvalues of selfadjoint exit space extensions via Weyl functions |
| 16:00 | Franciszek Szafraniec: The anatomy of matrices with unbounded operator entries |
| 16:30-17:00 | Sergey Korotov: Monotone matrices and discrete maximum principles in finite element analysis |

**Thursday**

| | |
|---|---|
| | *Chair:* Hans-Olav Tylli |
| 9:00 | Ville Turunen: Quantization of pseudo-differential operators on the 3-sphere |
| 9:30 | Fuad Kittaneh: Singular value inequalities for commutators of Hilbert space operators |
| 10:00 | Henk de Snoo: The Kato decomposition of quasi-Fredholm relations |
| 10:30 | Coffee |
| | *Chair:* Franciszek Szafraniec |
| 11:00 | Hans-Olav Tylli: Two-sided multiplication operators on spaces of bounded operators |
| 11:30 | Teresa Reginska: Ill-posed operator equations related to Cauchy problems for the Helmholtz equation |
| 12:00 | Olavi Nevanlinna: Polynomial numerical hull and construction of the resolvent operator |
| 12:30 | Lunch |
| | *Chair:* Rajendra Bhatia |
| 13:30 | Ludmila Nikolskaia: Modulus of continuity of an operator function |
| 14:00 | Miroslav Fiedler: Singular values and conditioning of matrices |
| 14:30 | Anne Greenbaum: Norms of functions of matrices |
| 15:00 | Coffee |
| | *Chair:* Rolf Stenberg |
| 15:30 | Alexander Gomilko, Jaroslaw Zemánek: Kreiss-type resolvent conditions |
| 16:00-16:30 | Ivan Oseledets: Some new results and algorithms for tensor-structured matrices in 3D problems |
| 18:00-22:00 | Conference Dinner at Saha |

**Friday**

| | |
|---|---|
| | *Chair:* Mikael Lindström |
| 9:00 | Rajendra Bhatia: Operator convex functions |
| 9:30 | László Zsidó: Multiple recurrence for $C^*$-dynamical systems |
| 10:00 | Marko Huhtanen: Approximate factoring of the inverse |
| 10:30 | Coffee |
| | *Chair:* Olavi Nevanlinna |
| 11:00 | Michael Ruzhansky: Pseudo-differential operators and symmetries |
| 11:30 | Jarmo Malinen: On a Tauberian condition for bounded linear operators |
| 12:00 | Concluding remarks |

# 4 Participants

## Local organizing committee

Mikko Byckling, Helsinki University of Technology
Antti Haimi, Helsinki University of Technology
Marko Huhtanen, Helsinki University of Technology
Olavi Nevanlinna, Helsinki University of Technology
Santtu Ruotsalainen, Helsinki University of Technology

## Speakers

Charles Batty, University of Oxford
Rajendra Bhatia, Indian Statistical Institute, New Delhi
Henk de Snoo, University of Groningen
Yu Farforovskaya, St. Petersburg University of Electrical Engineering
Miroslaw Fiedler, Academy of Sciences of the Czech Republic, Prague
Alexander Gomilko, National Academy of Sciences, Kiev
Anne Greenbaum, University of Washington, Seattle
Seppo Hassi, University of Vaasa
Marko Huhtanen, Helsinki University of Technology
Fuad Kittaneh, University of Jordan, Amman
Sergey Korotov, Helsinki University of Technology
Thomas Laffey, University College Dublin
Mikael Lindström, University of Oulu
Jarmo Malinen, Helsinki University of Technology
Ludmila Nikolskaia, Univérsité Bordeaux-1
Olavi Nevanlinna, Helsinki University of Technology
Ivan Oseledets, Institute of Numerical Mathematics, Moscow
Teresa Reginska, Polish Academy of Sciences, Warsaw
Michael Ruzhansky, Imperial College London
David Shoikhet, ORT Braude College, Karmiel
Franciszek Szafraniec, Uniwersytet Jagielloński, Krakow
Ville Turunen, Helsinki University of Technology
Hans-Olav Tylli, University of Helsinki
Jani Virtanen, University of Helsinki
Iwona Wróbel, Warsaw University of Technology

Jaroslaw Zemánek, Polish Academy of Sciences, Warsaw
Krystyna Zietak, Wroclaw University of Technology
László Zsidó, University of Rome 'Tor Vergata'

# 5 Abstracts

In alphabetical order.

# Rates of decay of smooth orbits of semigroups of operators

Charles Batty
University of Oxford
England

Thomas Duyckaerts
Université de Cergy-Pontoise
France

**Abstract**

A number of results are known showing that certain conditions on the resolvent of a bounded $C_0$-semigroup imply certain rates of decay for the smooth orbits of the semigroup. Such situations arise in the study of damped wave equations. We shall present a result of this type, which is both general and close to being sharp and which has a simple proof thanks to a device of Newman and Korevaar.

# Operator Convex Functions

Rajendra Bhatia
Theoretical Statistics and Mathematics Unit
Indian Statistical Institute
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**Abstract**

We present a new characterisation of operator convex functions on the positive half line very similar in spirit to Löwner's characterisation of operator monotone functions.

# The Kato decomposition of quasi-Fredholm relations

H.S.V. de Snoo
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**Abstract**

Quasi-Fredholm relations of degree $d \in \mathbb{N}$ in Hilbert spaces are defined in terms of conditions on their ranges and kernels. They are completely characterized in terms of an algebraic decomposition with a quasi-Fredholm relation of degree 0 and a nilpotent operator of degree $d$. The adjoint of a quasi-Fredholm relation of degree $d \in \mathbb{N}$ is shown to be quasi-Fredholm relation of degree $d \in \mathbb{N}$. The class of quasi-Fredholm relations contains the semi-Fredholm relations. Earlier results for quasi-Fredholm operators and semi-Fredholm operators are included [1], [2], [3]. This is joint work with J.-Ph. Labrousse (Nice), A. Sandovici (Piatra Neamt), and H. Winkler (Berlin).

## References

[1] I.C. Gohberg and M.G. Kreĭn, "The basic propositions on defect numbers, root numbers and indices of linear operators", Uspekhi Mat. Nauk., 12 (1957), 43-118 (Russian) [English translation: Transl. Amer. Math. Soc. (2), 13 (1960), 185-264].

[2] T. Kato, "Perturbation theory for nullity, deficiency, and other quantities of linear operators", J. d'Anal. Math., 6 (1958), 261-322.

[3] J.-Ph. Labrousse, "Les opérateurs quasi Fredholm: une généralisation des opérateurs semi Fredholm", Rend. Circ. Mat. Palermo (2), 29 (1980), 161–258.

# Modulus of continuity of an operator function.

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**Abstract**

**Theorem 1** *Let $A$ and $B$ be bounded selfadjoint operators on a separable Hilbert space. Suppose that the eigenvalues of $A$ and $B$ belong to the set $\nu_1, ..., \nu_n$ of real numbers such that $\nu_{i+1} - \nu_i = d > 0$. Let $f$ be a continuous function on an interval containing all points $\nu_i$. Then*

$$||f(A) - f(B)|| = \frac{2}{d} \max_{1 \le i \le n} |f(\nu_{i+1}) - f(\nu_i)| \sum_{k=1}^{n-1} \frac{\log(n-k)+1}{k} ||A - B||.$$

**Theorem 2** *Let $A$ and $B$ be bounded selfadjoint operators on a separable Hilbert space, $f$ be a continuous function on the interval $[a, b]$ containing the spectra of both $A$ and $B$. Denote $\omega_f$ the modulus of continuity of the function $f$. Then*

$$||f(A) - f(B)|| \le 4[\log(\frac{b-a}{||A-B||} + 1) + 1]^2 \cdot \omega_f(||A - B||).$$

To proof theorem 1 we prove some lemmas concerned Hadamard-Schur multipliers. In particular

**Lemma 3** *Let $T_k = (\frac{1}{k+|i-j|})_{i,j\ge 0}$, $k > 0$ be symmetric Toeplitz matrix determined by the sequence $(t_m)_{m\in\mathbb{Z}}$ : $t_m = \frac{1}{k+|m|}$. Then the matrix $T_k$ is a Hadamard-Schur multiplier and the multiplier norm of $T_k$ is : $||T_k||_{\mathcal{H}} = \frac{1}{k}$ .*

*Preprint submitted to Elsevier* *26 August 2008*

# Singular values and conditioning of matrices

Miroslav Fiedler
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**Abstract**

In the first part, some classical and some newer results on singular values and condition numbers of matrices will be presented.Then, a new parameter of a matrix will be introduced and its properties found.

# Kreiss-type Resolvent Conditions

Alexander Gomilko
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and
Jaroslav Zemánek
Institute of Mathematics
Polish Academy of Sciences
Warsaw, Poland

**Abstract**

We intend to discuss various Kreiss-type resolvent conditions, their mutual relations and connections with the behaviour to the powers of the operator in question. In particular, the strong Kreiss property implies the Cesàro boundedness of the powers [1].

## References

[1] A.M. Gomilko and J. Zemánek, On the uniform Kreiss resolvent condition, Funktsional. Anal. i Prilozhen. 42, no. 3, 81-84, (2008), in Russian.

# Norms of Functions of Matrices

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**Abstract**

Given an $n$ by $n$ matrix $A$, we look for a set $S$ in the complex plane and positive scalars $m$ and $M$ such that for all polynomials (or analytic functions) $p$, the inequalities

$$m \cdot \inf\{\|f\|_{\mathcal{L}^\infty(S)} : f(A) = p(A)\} \leq \|p(A)\| \leq M \cdot \inf\{\|f\|_{\mathcal{L}^\infty(S)} : f(A) = p(A)\}$$

hold, where $\|\cdot\|$ denotes the spectral norm. We try to find a set $S$ for which the ratio $M/m$ is of moderate size; e.g., much smaller than the condition number of an eigenvector matrix of $A$.

We show that for perturbed Jordan blocks – $n$ by $n$ matrices with ones on the superdiagonal and zeros elsewhere except for the $(n, 1)$ position, which is some $\nu \in (0, 1)$ – if $S$ is the unit disk, then $m = M = 1$. More generally, we show that for companion matrices whose eigenvalues lie in the open unit disk $\mathcal{D}$, one can take $m = 1$ if $S = \mathcal{D}$. We discuss the relationship of this work to Crouzeix's conjecture [1, 2] that $\|p(A)\| \leq 2 \max\{|p(z)| : z \in W(A)\}$, where $W(A)$ denotes the field of values of $A$. We prove Crouzeix's conjecture for perturbed Jordan blocks of dimension $n \geq 6$ and establish a constant 4 (but not yet 2) for perturbed Jordan blocks of sizes 3 through 5.

## References

[1] M. Crouzeix, *Numerical range and functional calculus in Hilbert space*, J. Functional Analysis 244 (2007), pp. 668-690.

[2] M. Crouzeix, *Bounds for analytical functions of matrices*, Integr. Equ. Oper. Theory 48 (2004), pp. 461–477.

# Characterization of eigenvalues of selfadjoint exit space extensions via Weyl functions

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**Abstract**

Eigenvalues of selfadjoint extensions in exit spaces are studied for symmetric operators in a Hilbert space with arbitrary defect numbers $(n, n)$, $n \leq \infty$. Both analytic and geometric criteria and various related characterizations are established.

The derivation of the main results and many of the given proofs rely on the notions of boundary relations and their Weyl families introduced in [1], and the general coupling technique developed very recently by Derkach, Hassi, Malamud and de Snoo; see [2, 3]. The general version of the coupling method needed here is a geometric approach for constructing exit space extensions for generalized resolvent and it provides an effective tool for studying spectral properties of selfadjoint exit space extensions via associated Weyl functions and their limiting behavior at the spectral points lying on the real axis.
The talk is based on a joint work with Mark Malamud (Donetsk).

## References

[1] V.A. Derkach, S. Hassi, M.M. Malamud, and H.S.V. de Snoo, "Boundary relations and Weyl families", Trans. Amer. Math. Soc., 358 (2006), 5351–5400.

[2] V.A. Derkach, S. Hassi, M.M. Malamud, and H.S.V. de Snoo, "Boundary relations and orthogonal couplings of symmetric operators", Proc. Algo-

rithmic Information Theory Conference, Vaasa 2005, Vaasan Yliopiston Julkaisuja, Selvityksiä ja raportteja, 124 (2005), 41–56.

[3] V. Derkach, S. Hassi, M. Malamud, and H. de Snoo, "Boundary relations and generalized resolvents of symmetric operators", arXiv, math.SP/0610299, (2006) 46 pp.

# Approximate Factoring of the Inverse

Marko Huhtanen
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**Abstract**

Let $\mathcal{W}$ and $\mathcal{V}_1$ be reasonably low dimensional sparse matrix subspaces[1] of $\mathbb{C}^{n\times n}$ over $\mathbb{C}$ (or $\mathbb{R}$) containing invertible elements. Assume that the nonsingular elements of $\mathcal{V}_1$ are readily invertible. Then, to approximately factor the inverse of a sparse nonsingular matrix $A \in \mathbb{C}^{n\times n}$ into the product $WV_1^{-1}$, consider the problem

$$AW \approx V_1 \tag{1}$$

with non-zero matrices $W \in \mathcal{W}$ and $V_1 \in \mathcal{V}_1$ regarded as variables both. Whether the equality holds is inspectable with the nullspace of the linear map

$$W \longmapsto (I - P_1)AW, \text{ with } W \in \mathcal{W}, \tag{2}$$

where $P_1$ is the orthogonal projection onto $\mathcal{V}_1$. There exists an exact factorization $AWV_1^{-1} = I$ with $V_1 = P_1AW = AW$ if and only if there are invertible elements $W$ in the nullspace. To have approximate solutions to

$$\inf_{W\in\mathcal{W},\, V_1\in\mathcal{V}_1} \left|\left|AWV_1^{-1} - I\right|\right| \tag{3}$$

in some norm of interest, sparse-sparse iterations are suggested for computing matrices $W$ and $V_1$ satisfying (1).

[1] A matrix subspace is sparse if its members are sparse with the same sparsity structure.

# Singular Value Inequalities for Commutators of Hilbert Space Operators

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**Abstract**

We prove several singular value inequalities for commutators of Hilbert space operators. It is shown, among other inequalities, that if $A$, $B$, and $X$ are operators on a complex separable Hilbert space such that $A$ and $B$ are positive, and $X$ is compact, then the singular values of $AX - XB$ are dominated by those of $\max(\|A\|, \|B\|)(X \oplus X)$, where $\|\cdot\|$ is the usual operator norm.

# Monotone Matrices and Discrete Maximum Principles in Finite Element Analysis

Sergey Korotov, Institute of Mathematics
Helsinki University of Technology, Espoo, Finland

**Abstract**

The talk is devoted to a preservation of qualitative properties of solutions of PDE models in numerical simulation. In [2, 3, 4] (see also references therein), it was shown how certain classes of monotone matrices can be used for providing one of such properties - the discrete maximum principle (DMP), in finite element (FE) calculations. In parallel, suitable (sufficient) geometric conditions on different shapes of finite elements for DMPs were formulated. However, in practice such geometric conditions are rather restrictive and often difficult to preserve during FE mesh adaptation [1]. Therefore we shall also discuss relevant weakening (algebraic and geometric) procedures.

## References

[1] Brandts, J., Korotov, S., Křížek, M., Šolc, J., On nonobtuse simplicial partitions, *SIAM Rev.*, to appear.

[2] Ciarlet, P. G., Raviart, P.-A., Maximum principle and uniform convergence for the finite element method, *Comput. Methods Appl. Mech. Engrg.* 2 (1973), 17–31.

[3] Hannukainen, A., Korotov, S., Vejchodský, T., Discrete maximum principles for FE solutions of the diffusion-reaction problem on prismatic meshes, *J. Comput. Appl. Math.* (in press), 1–16. Preprint 105(2008), Institute of Mathematics of the Academy of Sciences of the Czech Republic.

[4] Karátson, J., Korotov, S., Discrete maximum principles for finite element solutions of nonlinear elliptic problems with mixed boundary conditions, *Numer. Math.* 99 (2005), 669–698.

# The influence of appending zeros to the spectrum in the nonnegative inverse eigenvalue problem

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**Abstract**

Let $\sigma := (\lambda_1, ..., \lambda_n)$ be a list of complex numbers with

$$\lambda_1 = \max\{|\lambda_j| : j = 1, ..., n\}$$

and let

$$s_k := \lambda_1^k + ... + \lambda_n^k, \quad k = 1, 2, 3, \quad ...$$

be the associated Newton power sums.

A famous result of Boyle and Handelman (Annals of Mathematics 1991) states that if $\lambda_1 \neq \lambda_j \ (j > 1)$ and all the $s_k$ are positive, then there exists a nonnegative integer $N$ such that

$$\sigma_N := (\lambda_1, ..., \lambda_n, 0, ..., 0),$$

is the spectrum of a nonnegative $(n + N) \times (n + N)$ matrix $A$. The problem of obtaining a constructive proof of this result with an effective bound on the minimum number $N$ of zeros required has not yet been solved.

We present a number of techniques for constructing nonnegative matrices with given nonzero spectrum $\sigma$, and use them to obtain new upper bounds on the minimal size of such an $A$, for various classes of $\sigma$. For example, in the case $n = 3, \lambda_2 = \sqrt{-1}, \lambda_3 = -\sqrt{-1}$, $\sigma_N$ is the spectrum of a nonnegative matrix if and only if $\lambda_1 \geq \sqrt{\frac{2(N+3)}{N+2}}$.

This is joint work with Helena Šmigoc.

# Essential norm of operators on weighted Banach spaces of analytic functions

Mikael Lindström
Department of Mathematical Sciences
University of Oulu
Oulu, Finland

**Abstract**

In this talk we obtain an exact formula for the essential norm of any operator acting on weighted Banach spaces of analytic functions. The result has many applications to concrete operators acting on Bloch type spaces and on some weighted Bergman spaces.

## References

[1] P. Galindo and M. Lindström, Essential norm of operators on weighted Bergman spaces of infinite order, *J. Operator Theory*.

# On a Tauberian condition for bounded linear operators

Jarmo Malinen
Institute of Mathematics
Helsinki University of Technology

We consider the relation between the growth of sequences $\|T^n\|$ and $\|(n+1)(I-T)T^n\|$ for bounded operators $T$ satisfying variants of the Ritt resolvent condition $\|(\lambda - T)^{-1}\| \leq \frac{C}{|\lambda - 1|}$ in various subsets of $\{|\lambda| > 1\}$. We show that if $T$ satisfies the Tauberian condition

$$\sup_{n \geq 1} (n+1)\|(I-T)T^n\| \leq M < \infty \tag{1}$$

then a number of conditions are equivalent to power boundedness of $T$; namely $\sup_{n \geq 1} \|T^n\| \leq C < \infty$. Also weaker variants of (1) are discussed in the same context.

These results are a part of joint work with O. Nevanlinna, V. Turunen, and Z. Yuan; see [1, 2, 3].

## References

[1] J. Malinen, O. Nevanlinna, and Z. Yuan, On the Tauberian condition for bounded linear operators. To appear in: *Mathematical Proceedings of the Royal Irish Academy*, 2008.

[2] J. Malinen, O. Nevanlinna, V. Turunen, and Z. Yuan, A lower bound for the differences of powers of linear operators. *Acta Mathematica Sinica* **23**, 745–8, 2007.

[3] O. Nevanlinna, On the growth of the resolvent operators for power bounded operators. In *Linear Operators. Banach Center Publications* **38**, 247–64, 1997.

# Polynomial numerical hull and construction of the resolvent operator

Olavi Nevanlinna

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**Abstract**

Given any bounded operator $T$ in a Banach space $X$ we discuss algorithmic approaches to simple approximations for the resolvent $(\lambda - T)^{-1}$. The approximations are rational in $\lambda$ and polynomial in $T$. The approach is based on ideas related to polynomial numerical hull.

We link the convergence speed of the approximation to the Green's function for the outside of the spectrum of $T$ and give an application to computing Riesz projections.

The construction works as well in Banach algebras, where all Banach algebra operations are assumed to be available, except inverting elements.

Talk is based on report A 546 in http://math.tkk.fi/reports/.

**Related References**

[1] J. Burke, A. Greenbaum: Characterizations of the polynomial numerical hull of degree k, Lin. Alg. Appl. 419 (2006), pp. 37-47

[2] E.B. Davies: Spectral bounds using higher order numerical ranges, LMS J. Comput. Math. 8, 17-45 (2005)

[3] E.B. Davies: Linear Operators and their Spectra, Cambridge studies in advanced mathematics 106, Cambridge University Press (2007)

[4] Ch. Davis, A.Salemi: On polynomial numerical hull of normal matrices, Lin. Alg. Appl. 383 (2004) pp. 151-161

[5] V. Faber, A. Greenbaum, and D. E. Marshall, The polynomial numerical hulls of Jordan blocks and related matrices, Linear Algebra Appl., 374 (2003), pp. 231-246

[6] A. Greenbaum: Generalizations of the Field of Values Useful in the Study of Polynomial Functions of a Matrix, Lin. Alg. Appl. 347 (2002), pp. 233-249

[7] A. Greenbaum: Card Shuffling and the Polynomial Numerical Hull of Degree k SIAM J. Sci. Comput. 25 (2004), pp. 408-416

[8] P.R.Halmos: Capacity in Banach Algebras, Indiana Univ.Math. 20, pp.855-863 (1971)

[9] J. Korevaar: Green Functions, Capacities, Polynomial Approximation Numbers and Applications to Real and Complex Analysis, Nieuw. Arch. Wisk. (4) 4 (1986), pp. 133-153

[10] K.B. Laursen, M.M. Neumann: An introduction to local spectral theory, London Math. Soc. Monograph, 20, Oxford, Clarendon Press (2000)

[11] O.Nevanlinna: Convergence of Iterations for Linear Equations, Birkhäuser, Basel (1993)

[12] O.Nevanlinna: Hessenberg matrices in Krylov subspaces and the computation of the spectrum, Numer. Funct. Anal. and Optimiz., 16 (3,4), 443-473 (1995)

[13] O.Nevanlinna: Meromorphic Functions and Linear Algebra, AMS Fields Institute Monograph 18 (2003)

[14] O-P. Piirilä: Questions and notions related to quasialgebraicity in Banach algebras, Ann. Acad. Sci. Fenn. Math. Diss. 88 (1993)

[15] P. Tichý, J. Liesen: GMRES convergence and the polynomial numerical hull for a Jordan block, Preprint 34-2006, Institute of Mathematics, Technische Universität Berlin, 2006.

[16] P.Vrbová: On local spectral properties of operators in Banach spaces. Czechoslovak Math.J. 23 (98), pp 493-496 (1973)

# Some new results and algorithms for tensor-structured matrices in 3D problems

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**Abstract**

Suppose some integral or differential operator defined on $d$-dimensional cube is discretized The obtained matrix can be under very mild assumptions approximated by a sum of tensor products of matrices (tensor format). Tensor format provides great data compression, especially for high-dimensional problems, as well as the computational cost is reduced. Basic linear algebraic operations (summation of matrices, matrix multiplication) can be done very efficiently by using "one-dimensional" operations. By several research groups it was experimentally shown that matrix functions of a low-tensor rank matrices arising from mathematical physics can be approximated well in tensor format. However, no simple matrix tools for proving that fact are known. Recently we have discovered such a tool in two dimensions. The only case were the inverse of a low-tensor rank is also of low tensor rank that is known by now is a simple identity

$$(A \times B)^{-1} = A^{-1} \times B^{-1}.$$

By computer experiments it was found that for a matrix of a special form

$$A = I + X \times R_1 + R_2 \times Y,$$

where $R_1$ and $R_2$ are matrices of rank 1 the inverse $A^{-1}$ has tensor rank not higher than 5. This result can be generalized to matrices of

form

$$A = I + \sum_{i=1}^{r} X_i \times R_{i1} + R_{i2} \times Y_i,$$

where matrices $R_{i1}, R_{i2}$ are all of rank 1 then the tensor rank of $A^{-1}$ is bounded uniformly in $n$ by an estimate of form $\mathcal{O}(r^{\gamma})$. That means that the new class of structured matrices that is closed under inversion is found. This class is not very narrow and it can be proven that many practically important classes of matrices can reduced to it. Another question is the actual computation of matrix functions. For 3D case with the help of the celebrated Tucker format we were able to construct fast and efficient approximate inversion algorithm. For $256 \times 256 \times 256$ grid the inversion of the integral operator takes several minutes. Papers and codes may be obtained from the author by request or from *http://spring.inm.ras.ru/osel*. This work was partially supported by the RFBR grant 08-01-00115-a and the Priority Research Program of the Department of Mathematical Sciences of Russian Academy of Sciences.

## References


[1] I.V. Oseledets, E.E. Tyrtyshnikov, N.L. Zamarashkin, Matrix inversion cases with size-independent tensor rank estimates, *Linear Algebra Appl.*, 2008, submitted

[2] I.V. Oseledets, D.V. Savostyanov, E.E. Tyrtyshnikov, Linear algebra for tensor problems, *Computing* 2008, submitted

# Ill-posed operator equations related to Cauchy problems for the Helmholtz equation

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**Abstract**

In this talk we consider an ill-posed operator equation

$$Ax = y, \tag{1}$$

where $A : \mathcal{D}(A) \subset H \to H$ is a linear operator in Hilbert space. Under the term ill-posedness we mean that the solutions do not depend continuously on the data. The considered operator is related to the ill-posed boundary value problem for the Helmholtz equation under Dirichlet and Neumann conditions posed on a part $\Gamma$ of the boundary $\partial\Omega$ of $\Omega$. Such a problem is sometimes called a Cauchy problem for the Helmholtz equation. Uniqueness of a solution is shown for a general case when $\Gamma$ is a non-empty open subset of Lipschitz boundary $\partial\Omega$ (cf. [1]). Next, the problem of reconstructing a solution $x$ of (1) from inexact data $y^\delta$, $\|y - y^\delta\|$ is analyzed. We answer the following question: how to regularize the problem in such a way that the best possible accuracy is guaranteed? In the case when $\Omega$ is the infinite strip in $R^3$, we apply the spectral method given in [2]. The obtained order of convergence is compared with Tautenhahn's result based on [3].

# References

[1] W. Arendt, T. Regińska, An ill-posed boundary value problem for the Helmholtz equation on Lipschitz domains, *Ulmer Seminare* (2007).

[2] T. Regińska, K. Regiński, Approximate solution of a Cauchy problem for the Helmholtz equation, *Inverse Problems* (2006), 975-989.

[3] U. Tautenhahn, Optimality for linear ill-posed problems under general source conditions, *Num. Funct. Anal, and Optimiz.* 19 (1998), 377-398.

# Pseudo-differential operators and symmetries

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**Abstract**

The lecture is based on the joint work with Ville Turunen (HUT).

We study pseudodifferential operators globally on compact Lie groups and on compact manifolds. Instead of resorting to local coordinate charts we use the natural symmetries of the space. This approach allows us to introduce the notion of globally defined full symbols on compact manifolds and their symbolic calculus.

Let $G$ be a compact Lie group. The full symbol $a(x, \xi)$ can be viewed as a matrix valued function, with $x \in G$, $\xi \in \widehat{G}$, and symbol $a(x, \xi) \in \mathbf{C}^{\dim \xi} \times \mathbf{C}^{\dim \xi}$. There are also some structural conditions if one wants to ensure that the corresponding pseudo-differential operator belongs to the usual class $\mathrm{Op}S^m(G)$.

We will explain main ideas on the torus $\mathbf{T}^n$ and show how things can be generalised to compact Lie groups $G$. We will also review classes of compact manifolds on which this approach works, where we can impose a Lie group structure due to the geometric understanding related to the Poincaré conjecture.

Analysis on the torus appeared in [3] and [5], analysis on SU(2) appeared in [4] and [2], and the general analysis with further developments will appear in [1].

## References

[1] M. Ruzhansky and V. Turunen, Quantization of pseudo-differential operators and symmetries, in preparation, to appear in *Birkhäuser,* 2008/2009.

[2] M. Ruzhansky and V. Turunen, Global quantization of pseudo-differential operators on SU(2) and on the 3-sphere, preprint.

[3] M. Ruzhansky and V. Turunen, Quantization of pseudo-differential operators on the torus, arXiv:0805.2892v1

[4] M. Ruzhansky and V. Turunen, Pseudo-differential operators on group SU(2), arXiv:0802.2780v1

[5] M. Ruzhansky and V. Turunen, On the Fourier analysis of operators on the torus, Modern trends in pseudo-differential operators, 87-105, Oper. Theory Adv. Appl., 172, Birkhauser, Basel, 2007.

**Old and New in Complex Dynamics**
by

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**Abstract**

Historically, complex dynamics and geometrical function theory have been intensively developed from the beginning of the twentieth century. They provide the foundations for broad areas of mathematics. In the last fifty years the theory of holomorphic mappings on complex spaces has been studied by many mathematicians with many applications to nonlinear analysis, functional analysis, differential equations, classical and quantum mechanics. The laws of dynamics are usually presented as equations of motion which are written in the abstract form of a dynamical system: $\frac{dx}{dt} + f(x) = 0$, where $x$ is a variable describing the state of the system under study, and $f$ is a vector-function of $x$. The study of such systems when $f$ is a monotone or an accretive (generally nonlinear) operator on the underlying space has been recently the subject of much research by analysts working on quite a variety of interesting topics, including boundary value problems, integral equations and evolution problems .

There is a long history associated with the problem on iterating holomorphic mappings and their fixed points, the work of G. Julia, J. Wolff and C. Carathéodory being among the most important.

In this talk we give a brief description of the classical statements which combine celebrated Julia's Theorem in 1920 , Carathéodory's contribution in 1929 and Wolff's boundary version of the Schwarz Lemma in 1926 and their modern interpretations.

Also we present some applications of complex dynamical systems to geometry of domains in complex spaces and operator theory.

# The anatomy of matrices with unbounded operator entries

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**Abstract**

Closures and adjoints of operator matrices are confronted with those of matrix operators; this is taken out of [2]. As a kind of application an algorithm for subnormality of unbounded weighted shifts is considered. The later contributes to the everlasting desire of detecting unbounded subnormality and is in flavour of [1].

## References

[1] F.H. Szafraniec, On normal extensions of unbounded operators. IV. A matrix construction, *Oper. Theory Adv. Appl.*, **163** (2005), 337-350.

[2] M. Möller, F.H. Szafraniec, Adjoints and formal adjoints of matrices of unbounded operators, *Proc. Amer. Math. Soc.*, **136** (2008), 2165-2176.

# Quantization of pseudo-differential operators on the 3-sphere

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**Abstract**

This is a joint work with Michael Ruzhansky (Imperial College London). Symmetries of the 3-dimensional sphere $S^3$ enable a **global quantization** of pseudo-differential operators. This is an example of a more general procedure that can be carried out on compact Lie groups and homogeneous spaces. This gives another quantization for Hörmander's class of pseudo-differential operators on $S^3$, but now the **full symbol** can be globally defined. For a pseudo-differential operator

$$A : C^\infty(S^3) \to C^\infty(S^3),$$

the full symbol $\sigma_A(x,\xi)$ at point $x \in S^3$ and quantum number $\xi$ is a

$$(\mathbf{2}\xi + \mathbf{1}) \times (\mathbf{2}\xi + \mathbf{1}) - \mathbf{matrix}.$$

The symbol classes are characterized by simple global conditions.

## References

[1] M. Ruzhansky, V. Turunen, Quantization of pseudo-differential operators and symmetries, in preparation, to appear in *Birkhäuser,* 2008/2009.

# Two-sided multiplication operators on spaces of bounded operators

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**Abstract**

I will survey some recent results from [?], [?], [?] about quantitative properties of the basic multiplication operators $L_A R_B; S \mapsto ASB$, on the space $L(X)$ of bounded operators. Here $X$ is a classical Banach space, and $A, B \in L(X)$ are fixed operators. The results discussed will include: (i) $L_A R_B$ is strictly singular $L(L^p) \to L(L^p)$ for $1 < p < \infty$ if and only if $A$ and $B$ are strictly singular on $L^p$, (ii) $L_A R_B$ is strictly singular $L(L^p) \to L(L^p)$ for $p = 1, \infty$ if and only if $A$ and $B$ are weakly compact operators on $L^p$, (iii) $L_A R_B$ is weakly compact $L(L^p) \to L(L^p)$ for $2 < p < \infty$ if and only if either $A$ is compact, $B$ is compact, or $A \in \overline{G_{\ell^2}}$ and $JB \in \overline{G_{\ell^p}}$ for some isometry $J : L^p \to L^\infty$. (This last result from [?] answers a question from [?].) Here the operator $A \in L(L^p)$ belongs to the factorization ideal $G_{\ell^r}$ if $A = UV$, where $U \in L(\ell^r, L^p)$ and $V \in L(L^p, \ell^r)$.

## References

[1] W.B. Johnson and G. Schechtman, Multiplication operators on $L(L^p)$ and $\ell^p$-strictly singular operators, (preprint, 2007).

[2] M. Lindström, E. Saksman and H.-O. Tylli, Strictly singular and cosingular multiplications, *Canad. J. Math.*, 57 (2005), 1249-1278.

[3] E. Saksman and H.-O. Tylli, Weak compactness of multiplication operators on spaces of bounded linear operators, *Math. Scand.,* 70 (1992), 91-111.

# Norms of Toeplitz and Hankel matrices and their asymptotic behavior

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**Abstract**

We discuss the behavior of norms of finite Toeplitz matrices generated by Fisher-Hartwig symbols as the matrix dimension $n$ goes to infinity. In particular, we describe the asymptotics of the spectral norm of Toeplitz matrices as $n \to \infty$, which is of interest in time series with long range memory. We also mention the case of Schatten norms [1, 3], and consider similar questions for Hankel matrices.

## References

[1] A. Böttcher, Schatten norms of Toeplitz matrices with Fisher-Hartwig singularities, *Electron. J. Linear Algebra* **15** (2006), 251–259.

[2] A. Böttcher and J. A. Virtanen, Norms of Toeplitz matrices with Fisher-Hartwig symbols, *SIAM J. Matrix Anal. Appl.* **29** (2007), no. 2, 660–671.

[3] S.-W. Vong and X.-Q. Jin, Unitarily invariant norms of Toeplitz matrices with Fisher-Hartwig singularities, *SIAM J. Matrix Anal. Appl.* **29** (2007), no. 3, 850–854.

# On the Gauss-Lucas theorem, the numerical range and the Sendov conjecture

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**Abstract**

The Gauss-Lucas theorem states that the convex hull of the roots of a given polynomial contains the roots of its derivative. We will discuss the possibility of generalizing this result to the numerical range of companion matrices. We will also present several remarks on the conjecture posed by Blagovest Sendov concerning the location of roots of a polynomial and its derivative.

# On Some Known and Open Matrix Nearness Problems

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**Abstract**

A survey of matrix nearness problems is given in [1]. Difficulties of these problems first of all depend on selected norms which measure a distance of a given matrix from some given class of matrices. We use the unitarily invariant norms, in particular the $c_p$-Schatten norms and the spectral norm.

In the talk we develop the following matrix nearness problems:

- approximation of $A \in \mathcal{C}^{m \times n}$ by subunitary matrices with respect to any arbitrary unitarily invariant norm (see [2]),
- a minimal rank approximation of $A \in \mathcal{C}^{m \times n}$ with respect to the spectral norm (see [2]),
- approximation of $A \in \mathcal{C}^{n \times n}$ by matrices the spectrum of which is in a strip, with respect to the spectral norm (see [3]),
- strict spectral approximation of a matrix and some related problems (see [4], [5]).

We also discuss two open problems. The first problem is raised in [3] and the second one follows from [4], [5].

# References

[1] N.J. Higham, Matrix nearness problems and applications, in M.J.C. Gover, S. Barnett (eds), *Applications of Matrix Theory*, Oxford Univ. Press, 1989, 1–27.

[2] B. Laszkiewicz, K. Ziętak, Approximation of matrices and family of Gander methods for polar decomposition, *BIT Numer. Math.*, 46:345–366 (2006).

[3] B. Laszkiewicz, K. Ziętak, Approximation by matrices with restricted spectra, *Linear Algebra Appl.* 428:1031–1040 (2008).

[4] K. Ziętak, Strict approximation of matrices, *SIAM J. Matrix Anal. Appl.*, 16:232–234 (1995).

[5] K. Ziętak, Strict spectral approximation of a matrix and some related problems, *Applicationes Mathematicae*, 24: 267–280 (1997).

# Multiple recurrence for $C^*$-dynamical systems

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**Abstract**

Let $A$ be a $C^*$-algebra, $\varphi$ a state on $A$, and $\Phi$ a $*$-endomorphism of $A$, which leaves $\varphi$ invariant. Then the following recurrence property, which corresponds to the classical "Poincaré recurrence" for measure preserving transformations of finite measure spaces, always holds :

*for every* $0 \leq a \in A$ *with* $\varphi(a) > 0$ *we have*

$\varphi\Big(a \cdot \Phi^n(a)\Big) \neq 0$ *for some integer* $n \geq 1$.

Much less is known in the general setting about multiple recurrence, that is the validity, for a given integer $k \geq 2$, of the implication

$$0 \leq a \in A, \varphi(a) > 0 \implies$$
$$\varphi\Big(a \cdot \Phi^n(a) \cdot \Phi^{2n}(a) \cdot \ldots \cdot \Phi^{kn}(a)\Big) \neq 0 \textit{ for some integer } n \geq 1.$$

For commutative $A$, the validity of the above implication for every $k \geq 2$ is a deep multiple recurrence result of H. Furstenberg (see, for example, the book [1]). In this talk we discuss some results obtained in the case of arbitrary $A$ .

## References

[1] H. Furstenberg, Recurrence in Ergodic Theory and Combinatorial Number Theory, *Princeton Univ. Press,* 1981.