hp-Adaptive Finite Elements for Problems of Mechanics and Wave Propagation

Dates: June 2-6, 2008

Venue: Helsinki University of Technology (details later)

Format: 4 hours of lectures per day + 2 hours of computer lab (for those who want "hands on" exercise).

Lecturer: Dr. Leszek Demkowicz, Institute for Computational Engineering and Sciences, The University of Texas at Austin

Course coordinator: Antti H. Niemi (ahniemi@cc.hut.fi)

Evaluation: Based on participation + project work for those who choose to work with the code

Target group: Advanced master students, doctoral students and post-docs.

Prerequisites: basic mechanics and electromagnetism (linear acoustics, elasticity, Maxwell's equations), basic Functional Analysis (weak formulation of PDE's), basic numerical methods (Galerkin method, h-version of Finite Element Method), software engineering (Linux operating system, makefile, Fortran 90). Participants intending to learn the code should come with a LINUX laptop with installed Fortran 90 (we recommend Intel) and C compilers.

Course description:

Traditional, low order, finite element discretizations are well suited to resolve complex topologies and curvilinear geometries. The corresponding rates of convergence are limited by the polynomial order, and the regularity of the solution. Those include not only singularities coming from non-convex geometries and material interfaces but also regions with high gradients, (e.g. boundary layers) perceived by the computer in the preasymptotic range as singularities. In presence of problems with large geometrical or material contrasts, they "lock" (100 % error). For wave propagation problems, they suffer from large dispersion (phase) errors making solution of problems with large wave numbers impossible.

Spectral methods do not lock for singularly perturbed problems, and deliver exponential convergence, provided the solution is analytic up to boundary, i.e. no singularities are present on the boundary, They do not suffer from dispersion error for wave propagation. If the solution is, however, singular on the boundary or material interfaces, the advantage of using spectral methods is lost - the convergence slows down to algebraic rates again. They also behave very badly in the preasymptotic range if the meshes do not reflect well the structure of the solution. For complex curvilinear geometries, meshes are difficult to generate.

hp Finite Element Methods combine advantages of low order and spectral methods. The short course presents fundamental technological components of hp Finite Element Methods, including:

- Mathematical foundations (variational formulations, construction of H^1 and Hcurl-conforming elements, convergence estimates)
- Construction of hierarchical shape functions
- hp data structures
- Constrained approximation
- Geometry modeling. Exact geometry and isoparametric elements
- Projection-based interpolation

The hp technology culminates in a fully automatic hp-adaptive strategy in which element size h and polynomial degree p are automatically selected to construct a sequence of optimal meshes which deliver exponential convergence for both regular and singular solutions. The methodology is based on a coarse-fine grid paradigm where the fine grid solution guides optimal hp-refinements of the coarse grid. We shall discuss two versions of the hp-algorithm:

- The energy-driven hp-algorithm
- The goal-driven hp-algorithm.

The presentation will include a hands-on presentation of 1D and 2D hp codes, and a large number of 2D and 3D numerical examples, for both elliptic and Maxwell problems, focusing on wave propagation applications. For details on the subject, see [1,2].

We recommend getting a copy of [1] (comes with 1D and 2D hp codes). We will help to install the 1D and 2D codes on the participants' laptops. The laptops have to operate under LINUX and must have the Intel Fortran 90 and C compilers preinstalled.

[1] L. Demkowicz, ``*Computing with hp Finite Elements. I.One- and Two-Dimensional Elliptic and Maxwell Problems*'', Chapman & Hall/CRC Press, Taylor and Francis, October 2006.

[2] L. Demkowicz, J. Kurtz, D. Pardo, M. Paszynski, W. Rachowicz and A. Zdunek, ``*Computing with hp Finite Elements. II. Frontiers: Three-Dimensional Elliptic and Maxwell Problems with Applications*'', Chapman & Hall/CRC Press, Taylor and Francis, October 2007.

Topics to be covered in detail:

Day 1:

Review of model problems (linear acoustics, Maxwell equations,linear elasticity). Variational formulations, H^1,H(curl) and H(div) energy spaces, Galerkin method. Mesh Based Geometry (MBG) description.

Lab: Installing the codes

Day 2:

hp Finite Element Method: hierarchical shape functions, p- and h-adaptivity, 1-irregular meshes, constrained approximation

Lab: Preparation of geometry and initial mesh data input

Day 3:

hp data structures: basic objects and algorithms (natural order of elements, algorithms to determine neighbors and nodal connectivities including the information on local constraints)

Lab: Interfacing with linear solvers

Day 4:

Projection-Based Interpolation and automatic hp-adaptivity, examples Infinite Elements vs Perfectly Matched Layers

Lab: Exercising with the automatic hp-adaptivity

Day 5:

The exact sequence, H(curl)-conforming elements and Maxwell problems

Lab: Solving Maxwell equations,