Mini-Course: Numerical Integration with Splitting Methods

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Splitting methods constitute a class (numerical) schemes for solving initial value problems. Roughly speaking, they decompose the vector field into several parts and integrate these parts separately. Splitting methods are frequently used for the time discretisation of partial differential equations. The reason for this is that the splitting procedure yields highly competitive time stepping schemes which can dramatically reduce the required computational effort, compared to schemes based on the full vector field. Recent surveys on splitting methods are given in the monographs [1, 6] and in the article [8].

The first part of the mini-course will be devoted to splitting methods for ODEs. Following [1], we shall derive the (non-stiff) order conditions via the well-known Baker–Campbell–Hausdorff formula.

In the second part, emphasis will be laid on a sound convergence theory for splitting methods applied to time dependent partial differential equations. In particular, we will cover the following topics: evolution equations with unbounded operators; splitting methods for Schrödinger equations; dimension splitting for parabolic problems; boundary conditions; high-order splitting methods for analytic semigroups; nonlinear problems. The main reference for this part will be the article [7] and our recent papers [2, 3, 4, 5].

The mini-course will be complemented by practical exercises in Matlab.

References

[1] E. Hairer, C. Lubich and G. Wanner, *Geometric Numerical Integration. Structure-Preserving Algorithms for Ordinary Differential Equations*, 2nd ed., Springer, Berlin, 2006.

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[3] E. Hansen, A. Ostermann, *Exponential splitting for unbounded operators,* to appear in Math. Comp.

[4] E. Hansen, A. Ostermann, *Dimension splitting for quasilinear parabolic equations,* to appear in IMA J. Numer. Anal.

[5] E. Hansen, A. Ostermann, *High order splitting methods for analytic semigroups exist,* preprint, University of Innsbruck, 2008.

[6] W. Hundsdorfer and J. Verwer, *Numerical Solution of Time-Dependent Advection-Diffusion-Reaction Equations*, Springer, Berlin, 2003.

[7] T. Jahnke and C. Lubich, *Error bounds for exponential operator splittings*, BIT 40 (2000), pp. 735–744.

[8] R.I. McLachlan and G.R.W. Quispel, *Splitting methods*, Acta Numer. 11 (2002), pp. 341–434.