Perspectives in Inverse Problems

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Abstracts (as of 28th May 2004)

The Large Binocular Telescope (LBT): a laboratory for image restoration in astronomy

Mario Bertero

Università di Genova, Italy

LBT will consist of two 8.4 m mirrors on a common mount, with a spacing of 14.4 m between the centres of the two mirrors, so that a maximum baseline of 22.8 m will be available. First ligth is scheduled for October 16th, 2004. The images of the two mirrors will be combined interferometrically in a Fizeau mode by an interferometer called LINC/NIRVANA, the German-Italian beam combiner for LBT. Since, for a single image resolution is not uniform over the field, several images of the same astronomical object must be acquired with different orientations of the baseline and they must be suitably processed in order to get a unique image with a uniform resolution over the field. Therefore imaging by LBT will require routinely the use of multiple-images deconvolution methods if the goal is a unique image with the resolution of a 22.8 m telescope.

In this talk we present a general approach to the design of iterative methods for the simultaneous deconvolution of multiple images; they apply to the minimization of functionals derived from maximum likelihood and/or bayesian (regularized) formulations of the problem, including Poisson (photon counting) or Gauss statistics of the noise. Moreover several regularization terms can be considered, including Tikhonov, maximum entropy and others. The nice feature of the approach is that all algorithms can be easily implemented and tested. Moreover it is possible to consider accelerated versions by exploiting the block structure of the matrix. The approach is derived from recent work of Lanteri and coworkers.

New methods in tensorial tomography for medicine and seismology

Alexander A. Boukhgueim

Schlumberger, Norway

The talk is devoted to the theory and numerical solution of a number of planar tomography problems and their applications to the solution of inverse kinematic problems of seismology. The aim of this work was the derivation of new inversion formulae and their effective numerical implementation. We derive new inversion formulae for the emission tomography problem (for the scalar, vector and tensor cases). In the vector case it becomes possible to reconstruct the full vector field (and not merely its solenoidal part like in the unattenuated case), provided that the attenuation function doesn't vanish. We also obtained a singular value decomposition of the Radon transform of tensor fields in the framework of the fan-beam scanning geometry, it allows to characterize the range of the tensorial Radon transform, invert it and estimate the level of incorrectness. Then we consider several statements of the inverse kinematic problem of seismology, choose the stable ones and derive inversion formulae for the case of reflected rays (in a 2D layer) and for the case of refracted rays (in a 3D volume). In the case of refracted rays, a Newton-type algorithm for finding the 3D velocity distribution from 3D travel time measurements is constructed. As a first approximation we choose a sound velocity that increases linearly with the depth. With this choice for the linearization, the underlying problem reduces to a sequence of 2D Radon transforms in discs.

This is joint work with Alexander L. Boukhgueim and Sergei Kazantsev.

Volume estimates via boundary distance function, a problem of Busemann, and a generalized Minkowski existence theorem

Dmitri Burago

Penn State University, USA

We are going to discuss the following topics, which happen to be closely related:

1. What are the restrictions on the Gaussian image of a closed surfaces (or of surfaces with planar boundaries)? To be more precise, we are interested in weighted Gaussian images, that is in the images of surfaces area under the Gauss map (and the push forward of the surface area under this map). For instance, one wonders: under what conditions can one construct a two-dimensional polyhedral surface in $\mathbb{R}^4$ with given areas and directions of faces and no boundary (or a boundary lying in a given two-dimensional plane)? That is, given a set of k-vectors, can we "build a roof" whose faces correspond to the k-vectors? In co-dimension 1 this question is answered by the classic Existence Theorem of Minkowski.

2. Do flats minimizer surface area among all surfaces with the same boundary? Of course, they do for the surface area functional induced by Euclidean structure, but what about a normed space (a problem posed by Busemann about 50 years ago)? A related (and more general) question is: what is the relationship between different types of ellipticily for general surface area functionals (an area functional is said to be elliptic over Z (resp. R) if regions in affine planes are area minimizers among all Lipschits chains over Z (R) with the same boundary).

3. Optimal fillings: metrics on a manifold (with boundary) that admit no volume decreasing perturbations that do not decrease distances between boundary points. In other words, we will be looking for situations when an inequality for boundary distance functions implies corresponding inequality for the volumes of the metrics.

4. Asymptotic growth of volume for large balls in a periodic metric (a metric that admits a co-compact isometric action of an abelian group). In other words: what is approximately the area of a ball of a large radius R in the universal cover of a Riemannian (or even Finsler) torus?

This is a joint project with Sergei Ivanov.

Mixed boundary value problems in inverse scattering

Fioralba Cakoni

University of Delaware, USA

Mixed boundary value problems in electromagnetic scattering theory arise when the scattering object is a composite material such that parts of the scatterer have different electrical properties. In general the physical properties of the scattering object are not known a priori, e.g. it is not known if an object is coated or not and if so what the extent and the composition of the coating is.

The inverse scattering problem we will discuss in details is to determine the shape and the surface conductivity of a anisotropic dielectric that is partially coated with a thin layer of highly conductive material from a knowledge of the asymptotic behavior of the scattered field due to the scattering of an incident time-harmonic electromagnetic plane wave at fixed frequency. General issues concerning the direct and inverse scattering problem for mixed boundary value problems and related open questions will be addressed.

Aristotelian prior boundary conditions

Daniela Calvetti

Case Western Reserve University, USA

The selection of boundary condition when restoring a finite 1D or 2D signal has received a lot of interest in recent times. Dirichlet, periodic, reflective and antireflective boundary conditions may be appropriate selection for some problems, while wholly unreasonable for others. In this paper we show that, ina an Aristotelian approach to knowledge, when it is not known a-priori which boundary conditions should be chosen, by admitting our ignorance, it is possible to let the data itself determine them. Aristotelian boundary conditions in the deblurring of 1D and 2D finite signals with Tikhonov regularization or truncated iterative methods is also discussed. Computed examples showing the effectiveness of Aristotelian boundary conditions are presented.

New multiscale thoughts on limited-angle tomography

Emmanuel Candes

Caltech, USA

This talk is concerned with the problem of reconstructing an object from noisy limited-angle tomographic data—a problem which arises in many important medical applications. Here, a central question is to describe which features can be reconstructed accurately from such data and how well, and which features cannot be recovered.

We argue that curvelets, a recently developed multiscale system, may have a great potential in this setting. Conceptually, curvelets are multiscale elements with a useful microlocal structure which makes them especially adapted to limited-angle tomography. We develop a theory of optimal rates of convergence which quantifies that features which are microlocally in the "good" direction can be recovered accurately and which shows that adapted curvelet-biorthogonal decompositions with thresholding can achieve quantitatively optimal rates of convergence. We hope to report on early numerical results. This is joint work with David Donoho.

Can one find the structure of a mixture from its effective properties?

Elena Cherkaev

University of Utah, USA

The problem of characterizing the microstructure of a composite material is reduced to reconstruction of the spectral measure that contains all information about the structure. The spectral function can be uniquely recovered from the Stieltjes representation using effective complex permittivity of the composite material measured in an interval of frequency. Self-adjointness of the operator is used to formulate a stable reconstruction problem for the non-decreasing spectral measure. Classes of composites are discussed that admit unique reconstruction of microgeometry.

Open problems in inverse electromagnetic scattering theory

David Colton

University of Delaware, USA

We discuss some recent work done at the University of Delaware on uniqueness theorems and reconstruction algorithms for inverse electromagnetic scattering theory with particular concern for anisotropic media and mixed boundary value problems.Open problems arising from this work will be mentioned.

Computational issues of inverse problems in film restoration: some experiences

Luisa d'Amore and Almerico Murli

Università di Napoli, Italy

We will review on an ongoing research activity regarding the automatic restoration of old black and white movies.

Movies restoration is a very difficult problem, first because of the many types of degradation. Here we discuss the design of numerical algorithms for detection and removal of "blotches". The main idea for detecting and removal such kind of defects is to use information given by the movement in the sequence. Unfortunately, we are not able to compute the 2D motion field, that is the bidimensional projection of the true tridimensional movement, but only a an apparent motion, which is usually called as "optical flow".

The removal of blotches and the computation of the optical flow are both inverse and ill posed problems. In order to preserve discontinuities, the regularization leads to non linear Euler-Lagrange equations.

The main contribution of our work is to use a particular spatio - temporal interpolation for removing blotches, and the use of the multiresolution approach for a reliable estimate of the velocity field, based on a gaussian generating function. Moreover, our focus is on difficulties due to the high computational requirements, which can be addressed by exploitation of advanced computing resources.

Factorization of seismic inverse scattering operators

Maarten de Hoop

Colorado School of Mines, USA

To detect heterogeneities in Earth's interior that are singular in nature, the use of scattered body wave phases is pertinent. The singular parts of these waves contain information about non-smooth variations in material and physical properties of the Earth. The singular part of scattered body waves is described by microlocal analysis. Inverse scattering of these waves has been formulated and carried out in terms of Fourier integral operators and their clean intersection calculus.

Dyadic-parabolic decomposition applied to Fourier integral operators appears as a promising tool in seismic inverse scattering, both from an observational perspective as well as from a computational perspective. Scattered phases relevant to the study of upper mantle discontinuities (associated with phase transitions), for example, are usually observed in relatively small arrays of stations; such small arrays can be accommodated in the seismic inverse scattering operator, in principle, by a parabolic cutoff. On the other hand, the work by H. Smith (1997) shows the potential of the decomposition in finding sparse representations of the inverse scattering operator. In this context, the frame of second generation curvelets introduced by Candes and Guo (2002) can be shown to yield such a sparse representation (Candes and Demanet, 2002).

A problem to be overcome are the conditions on the Fourier integral operator for the decomposition to be applicable. In particular, the condition that the canonical relation is a local canonical graph seems an obstruction for the seismic inverse problem (in the presence of caustics). In this presentation we show ways to factorize the inverse scattering operator into component operators which satisfy this condition.

Symplectic Field Theory and commuting differential operators

Yakov Eliashberg

Stanford University, USA

Symplectic Field Theory provides an approach to the theory of holomorphic curves in the spirit of a topological field theory. Algebraic structures arising in this theory shed some new light on connections between Gromov-Witten theory and integrable systems.

Regularization in neural networks and fuzzy control, stochastic convergence concepts

Heinz Engl

Radon Institute for Computational and Applied Mathematics, Austria

This talk is devoted to some quite non-standard, loosely connected applications of and concepts for regularization methods: First, we will talk about ill-posed and convergence (with rates) of regularization methods for the closely related problems of neural network training and the construction of Sugeno controllers, viewed as nonlinear inverse problems. Then, we will outline a concept and present first results, still for linear ill-posed problems, for obtaining convergence in distribution (with rates in the Prokhorov metric) of regularization methods for problems where the errors are modelled in a stochastic way. The key is the combination of traditional (functional analysis based) convergence rates results with a-priori assumptions about the probability of large errors, using a quite old "lifting" result by Engl and Wakolbinger. In addition to the fact that stochastic error concepts are important especially in connection with neural networks and fuzzy control, a methodological link between the two parts of this talk is the use of source conditions.

Factorization of S-matrix for many dimensional Schroedinger operator

Ludwig Faddeev

Steklov Mathematical Institute, Russia

The solution of the inverse scattering problem involves the factorisation of the scattering matrix, which is used for the construction of the weight operator, entering the Gelfand-Levitan-Marchenko equation. The concrete form of such factorisation for many dimensional Schroedinger operator is presented. The analiticity properties of the corresponding kernels is discussed.

Computing statistical solutions to physical inverse problems

Colin Fox

University of Auckland, New Zealand

Algorithms that sample an arbitrary probability distribution provide the solution, up to computational efficiency, of practical inverse problems that can be mathematically specified, i.e. where data is measured via a specified forward map with characterized noise statistics. These algorithms are provably convergent to any desired statistic of the posterior distribution, and hence can answer any question about the unknown image that is sought from the measured data.

In physical inverse problems, computation of the forward map requires simulation of a sophisticated physical model. Then it would seem that the computations required are prohibitive and convergence times might rival the lifetime of the universe. Actually, recent theoretical and computational advances have yielded algorithms that compute summary statistics over useful time scales while employing forward maps that are calibrated against real measurements. Near real-time solutions using these algorithms appear to be within reach. We outline recent algorithms and also the outstanding theoretical issues that couldprovide further algorithmic efficiencies.

Pointwise boundary determination of the conductivity from the local Dirichlet-to-Neumann map

Romina Gaburro

Università degli Studi di Trieste, Italy

In absence of internal sources the conductivity equation of a body, described by a domain $\Omega \subset \mathbb{R}^n$, is $\mathrm{div}(A\nabla u) = 0$ in Ω, where the electrostatic conductivity is represented by the symmetric, positive definite matrix $A = A(x)$, $x \in \Omega$ and u is the electrostatic potential in Ω. In this talk we will address the inverse (anisotropic) conductivity problem of finding A when the so called Dirichlet-to-Neumann map is locally given on a non empty portion Γ of the boundary $\partial\Omega$. We will in particular consider the question of whether it is possible to determine pointwise the conductivity at the boundary and we will give a pointwise stability result at the boundary. We will finally discuss whether it is possible to reconstruct the conductivity by making use of singular solutions of the conductivity equation having a singularity $\bar{x}$ outside Ω, with $dist(\bar{x}, \partial\Omega)$ small as much as we wish.

Inverse problems for the Schrodinger operator in a layer

Patricia Gaitan

Université de Provence, France

Let H_0 be the Laplace operator with Dirichlet conditions in $L^2(\Omega)$, $\Omega = \mathbb{R}^n \times (0, \pi)$, $n \geq 2$, V being a perturbation such that $|V(x, y)| \leq C(1 + |x|)^{-\theta}$, $(x, y) \in \Omega$, $C > 0$, $\theta > 1$ and $H := H_0 + V$. We give an appropriate definition of the scattering amplitude. We prove that the scattering operator associated to (H, H_0) uniquely determines the potential V. Moreover, if $(\lambda_q)_{q \geq 1}$ is a sequence of energies which tends to infinity and whose position as regards the thresholds of H depends on the asymptotic behaviour of V when $|x| \to \infty$ ($\theta > n$, respectively exponentially decreasing), we prove that the scattering amplitudes in λ_q, for $q \geq 1$, determine uniquely V.

Bi-isospectral pentadiagonal oscillatory matrices

Kazem Ghanbari

Sahand University of Technology, Iran

For a given pentadiagonal Oscillatory matix A, we find a family of other pentadiagonal Oscillatory matrices having two spectrum in common with A.

Microlocal methods for the Calderon problem

Allan Greenleaf

University of Rochester, USA

I will discuss results concerning the recovery of a potential or conductivity from boundary measurements of solutions of the corresponding operator. The techniques are adapted from microlocal analysis and are useful when the unknown potential or conductivity is piecewise smooth, or more generally, conormal with respect to a smooth submanifold.

Tikhonov regularization for the inverse problem of option pricing in the price-dependent case

Torsten Hein

Chemnitz University of Technology, Germany

The talk deals with analytic studies for solving the inverse problem of identifying a price-dependent volatility from given strike-dependent option price data. The forward operator is formulated as a mapping between the Hilbert Spaces $H^1(R)$ and $L^2(R)$ and properties of this operator are discussed. It is shown that the well-known theory of nonlinear Tikhonov regularization is applicable to this problem.

Complex Lagrangian tori and spectral asymptotics for non-selfadjoint operators

Michael Hitrik

UCLA, USA

Following a work by A. Melin and J. Sjöstrand, it has become increasingly clear that non-selfadjoint operators in dimension two share many of the pleasant features of operators in dimension one. In the semiclassical limit, it is frequently possible to give complete asymptotics for individual eigenvalues of such operators in some domains in the complex spectral plane. In this talk, we would like to discuss some results obtained in this direction together with Johannes Sjöstrand, as part of an ongoing program on small non-selfadjoint perturbations of selfadjoint operators. In particular, we hope to discuss the recent results in the case when the classical flow of the unperturbed part admits invariant Lagrangian tori satisfying a Diophantine condition.

Inverse eigenvalue problems and inverse scattering

Miklós Horváth

Budapest University of Technology and Economics, Hungary

Consider the Schrödinger equation $-y'' + q(x)y = \lambda y$ on a finite interval or on the half-line $[0, \infty)$. If we impose boundary conditions in both endpoints, the resulting set of eigenvalues does not determine the potential $q(x)$. If, on the other hand, we apply varying boundary conditions in one of the endpoints, we can characterize which sets of eigenvalues define the potential uniquely. We also investigate the three-dimensional inverse scattering problem with fixed energy and describe how many phase shifts are needed to define the potential. The two problems are shown to be closely related.

Inverse spectral problems for Sturm–Liouville operators with singular potentials

Rostyslav Hryniv

Universität Bonn, Germany

The talk is based on a joint project with Yaroslav V. Mykytyuk (Lviv National University).

Suppose that σ is an arbitrary real-valued function from $L_2(0,1)$, put $q = \sigma' \in W_2^{-1}(0,1)$, where the derivative is understood in the distributional sense, and consider the Sturm-Liouville operator T with potential q given formally by $Tf = -f'' + qf$ and, say, the Dirichlet boundary conditions. It was proved in the works by Savchuk and Shkalikov [Sturm-Liouville operators with singular potentials, *Matem. Zametki (Math. Notes)* **66**(1999), no. 6, 897–912] that T can be naturally and uniquely defined to give a selfadjoint bounded below operator in $L_2(0,1)$ with discrete spectrum $\{\lambda_k^2\}$, $k \in \mathbb{N}$. Moreover, λ_k were shown [Savchuk A. M., On eigenvalues and eigenfunctions of Sturm-Liouville operators with singular potentials, *Matem. Zametki (Math. Notes)* **69**(2001), no. 2, 277–285] to satisfy the asymptotics $\lambda_k = \pi k + \mu_k$ for some ℓ_2-sequence (μ_k).

We answer the question which sequences $\{\lambda_k^2\}$ with the above asymptotics are indeed spectra of Sturm-Liouville operators with singular potentials from the class $W_2^{-1}(0,1)$, i.e., solve the inverse spectral problem for this class of operators. We show how to recover the potential q given the Dirichlet spectrum of T and, say, the sequence of norming constants or a Dirichlet-Neumann spectrum of T and describe explicitly the corresponding set of spectral data.

All other classical inverse spectral problems can also be solved in the set of Sturm-Liouville operators with potentials from the space $W_2^{-1}(0,1)$.

The probe and enclosure methods
— two methods in inverse problems

Masaru Ikehata

Gunma University, Japan

The probe and enclosure methods are general ideas for extracting information about unknown objects (obstacles, inclusions, cracks etc.) embedded in a known background medium from the data given by the Dirichlet-to-Neumann map or its partial knowledge at the boundary of the medium. They have been applied to the inverse scattering problems and boundary value problems. We describe the core of the methods to recent applications together with some open problems.

Detecting corrosion on inaccesible interfaces

Gabriele Inglese

CNR-IAC section of Firenze, Italy

We consider the problem of detecting corrosion on the inaccessible interface between a metallic plate and a medium (air, water, ...). Data (ex: temperature, heat flux density) are collected in an accessible side of the boundary of the plate. Temperature inside the plate is assumed to satisfy Laplace's equation. We suppose that the effects of corrosion attack consist in some material loss and (alternatively or in addition) in a perturbation of the Robin boundary condition that usually models energy transfer through the interface. These models developed in the first '90s. Literature about them include papers by Santosa, Vogelius, Brian, Jaoua, Ben Abda, Alessandrini and many others.

Problems in electrical impedance imaging
of heart disease and breast cancer

David Isaacson

Rensselaer Polytechnic Institute, USA

We explain how problems in monitoring heart and lung function as well as diagnosing breast cancer lead naturally to inverse problems for Maxwell's equations. Since the electrical conductivity of heart and lungs vary as the volumes of blood and air within them vary images of the conductivity inside the body may be used to monitor ventilation and perfusion. Since the electrical conductivity of breast tumors is significantly higher than surrounding normal tissue images of the electrical conductivity within the breast may be used to diagnose breast cancer.

We explain how the design of systems to image the electrical conductivity and permittivity inside a body from measurements on the body's surface gives rise to problems in the spectral analysis of Neumann to Dirichlet maps. The solutions to some of these problems will be explained and illustrated with images and movies of heart and lung function made by the RPI Adaptive Current Tomography system (ACT3).

Inverse problems for the elasticity systems

Victor Isakov

Wichita State University, USA

We consider the classical dynamical elasticity system and we are interested in recovery time independent coefficients of this system from boundary measurements. We report on recent global uniqueness and stability results in this inverse problem obtained by various techniques, including Carleman type estimates, and on related topics in uniqueness of the continuation and exact and approximate controllability of this system.

Inverse problems and hyperbolic spaces and an application to local DN map

Hiroshi Isozaki

University of Tsukuba, Japan

We present a new approach to the inverse problem based on hyperbolic manifolds. There are 4 main issues:

- Solving the inverse boundary value problems in $\mathbf{R}^n (n \geq 3)$ by using hyperbolic manifolds
- Local identifiability of conductivities from the local DN map
- Detection of location of inclusions from the local DN map
- $\overline{\partial}$-theory on $\mathbf{H}^3$

The starting point is the following observation. Given a boundary value problem $(-\Delta + q)u = 0$ in $\Omega \subset \mathbf{R}^n$, let $v = x_n^{(2-n)/2}u$. Then v satisfies $(-\Delta_g + V)v = 0$, where $V = x_n^2 q - n(n-2)/4$ and $\Delta_g = x_n^2\partial_n^2 - (n-2)x_n\partial_n + x_n^2\Delta_x$, which is just the Laplace-Beltrami operator on $\mathbf{H}^n$. (We denote the points in $\mathbf{R}^n$ by $(x, x_n), x \in \mathbf{R}^{n-1}$, and the domain Ω is regarded to sit in the region $\{x_n > 0\}$). This means that the inverse boundary value problem in $\mathbf{R}^n$ and the one in $\mathbf{H}^n$ are equivalent. To study the latter, the main tool we use is the Green function of the gauge-transformed operator $H_0(\theta) = -e^{-ix\cdot\theta}\Delta_g e^{ix\cdot\theta}, \theta \in \mathbf{C}^{n-1}$, which is written in terms of modified Bessel functions: $e^{i(x-x')\cdot\xi}(yy')^{n-1)/2}I_\nu(\zeta(\xi,\theta)y)K_\nu(\zeta(\xi,\theta)y), \zeta(\xi,\theta) = \sqrt{(\xi+\theta)^2}$. This is a counter part of Faddeev's Green function $\int e^{i(x-x')\cdot\xi}(\xi^2 + 2\zeta\cdot\xi)^{-1}d\xi$, a fundamental tool in the inverse scattering theory for Schrödinger operators on the Euclidean space.

Inverse boundary value problems. To solve the inverse boundary value problem in $\mathbf{H}^n$, we pass it to the quotient space $\mathbf{H}^n/\Gamma$, where Γ is a lattice in $\mathbf{R}^{n-1}$ of rank $n-1$ which is taken so large that the fundamental domain $E = \mathbf{R}^{n-1}/\Gamma$ contains the region Ω completely inside. In this case, θ varies over the fundamental domian of the dual lattice of Γ and is regarded as a Floquet parameter in the theory of periodic Schrödinger operators. >From the Dirichlet-Neumann map for the boundary value problem, one can construct the scattering amplitude. This latter is analytic in some region of complexified Floquet parameters θ. By passing to this parameter to infinity (complex Born approximation), one can reconstruct q from the DN map.

Local DN map and its applications. For a domain $\Omega \subset \mathbf{R}^3$, consider the equation of electric conductivity: $\nabla\cdot(\gamma(x)\nabla u) = 0$. Suppose that $\gamma(x)$ is sufficiently smooth and let Λ_γ be the DN map. Take x_0 from the boundary of the convex hull of Ω. Then if $\Lambda_{\gamma_1} f = \Lambda_{\gamma_2} f$ on $B(x_0,R)\cap\partial\Omega$ for any $f \in H_0^2(B(x_0,R)\cap\partial\Omega)$, where $B(x_0,R)$ is the ball of radius R with center at x_0, then $\gamma_1(x) = \gamma_2(x)$ on $B(x_0,R)\cap\Omega$. The proof relies on the construction of solutions to the equation $\nabla\cdot(\gamma\nabla u) = 0$ whose support concentrates on a plane, and also uses isomteries on $\mathbf{H}^3$ which converts the plane to a hemi-sphere centered at $\{x_3 = 0\}$. This is a joint work with G. Uhlmann.

This idea can also be used to detect the location of non-smooth part of conductivities. Suppose for the sake of simplicity that we know the DN map Λ_0 for the case that γ is a constant, and that the conductivity is different from this constant on a subset $\Omega_1 \subset \Omega$. Take x_0 from outsied of the convex hull of Ω and let $S_{out}^\epsilon = \{x \in \partial\Omega; |x - x_0| > R + \epsilon\}, S_{in}^\epsilon = \{x \in \partial\Omega; |x - x_0| < R - \epsilon\}$. Then one can construct the boundary data $f_\tau(x)$ depending on a large paramter $\tau > 0$ having the following properties: On S_{out}^ϵ (S_{in}^ϵ), $f_\tau(x)$ is exponentially decreasing (increasing) in τ. Let Λ be the DN map for γ. If $R < dis(x_0,\Omega_1)$, then $0 \leq (\Lambda - \Lambda_0)f_\tau, f_\tau) < Ce^{-\delta\tau}$, and if $R > dis(x_0,\partial\Omega_1)$, then $((\Lambda - \Lambda_0)f_\tau, f_\tau) > C'e^{\delta\tau}$. This means that one can detect the location of inclusions from the boundary data which are essentially localized on a part of the boundary.

$\overline{\partial}$-*approach*. One can define an analogue of Faddeev's scattering amplitude for $-\Delta_g + V$ on $\mathbf{H}^3$ depending on a parameter $\theta \in \mathbf{C}^2$. As in the Euclidean case, this scattering amplitude satisfies a $\overline{\partial}$-equation and admits an integral representation of the potential.

References

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[IsUh] H. Isozaki and G. Uhlmann. *Hyperbolic geomtry and local Dirichlet-Neumann map*, to appear in Adv. in Math..

Reconstruction of kernels in parabolic and hyperbolic integro-differential equations from restricted Dirichlet-to-Neumann operator

Jaan Janno

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We consider inverse problems to determine time- and space-dependent kernels in parabolic and hyperbolic integro-differential equations. We show that these kernels are recovered by Dirichlet-to-Neumann operators of the problems restricted to Dirichlet data of the form of products of a fixed time-dependent and arbitrary space-dependent functions.

Inverse problems in astronomy

Mikko Kaasalainen

University of Helsinki, Finland

I discuss some inverse problems in astronomy and astrophysics. One important area is remote sensing in solar system studies. Detailed models of asteroids, the largest and least well known object population of our solar system can be constructed using photometric and complementary data. We have coined the term "multidatainversion" for the simultaneous inclusion of interferometric, radar (pure Doppler or also range-resolved), adaptive optics, and stellar occultation data. Multidatainversion will also be used in building models of the planets and their orbits in solar systems other than our own.

Another topical area is the building of self-consistent dynamical models of our galaxy, a complex dynamical system of some 100 billion gravitationally interacting stars. This system can be described by suitable distribution functions in phase space, and these functions must be adjusted to fit the observed stellar positions and velocities while imposing the constraint that they satisfy the dynamical equations describing a physically self-consistent large gravitational system.

Inverse problems, approximation errors and inverse crimes

Jari P. Kaipio and Erkki Somersalo

University of Kuopio and Helsinki University of Technology, Finland

We discuss the approximation of computational observation and prior models from the infinite-dimensional ones. An inverse crime is a numerical evaluation approach in which the same computational model is used in both the forward and inverse computations. It is widely known that by committing an inverse crime, unrealistically positive results for inversion methods can be obtained. We discuss the relevance of inverse crimes and approximation errors and show how to cope with the latter ones with the statistical inversion approach.

Gaussian beams in inverse problems

Alexander Katchalov

Steklov Mathematical Institute, Russia

Inversion formula for the Radon transform on strip with fan beam data

Daniel Kazantsev

Institute of Computational Mathematics and Mathematical Geophysics of SB RAS, Russia

This work tackles the problem of two - dimensional image reconstruction and inversion formula of the Radon transform of functions defined in rectangular domain (strip-like) within the framework of the data model using fan-beam collimators.

The characteristics of this transform are described and algorithm of back projection type with filtering using the Hilbert transformation is proposed. Another approach using the Fourier transform is considered and inversion formula has been derived. For computer modeling we have developed codes and graphical user interface in Matlab, investigated various parameters, such as a number of receiver/source pairs, their geometrical positions, background and test objects density, angle, etc.

Numerical experiments with test objects in the case of incomplete data (limited angle) are performed and results of comparison of the two derived algorithms are illustrated.

Statistical inversion for 3D x-ray tomography with sparse data

Ville Kolehmainen

University of Kuopio, Finland

In x-ray tomography the structure of a three dimensional body is reconstructed from a collection of projection images of the body. In many practical applications, only a few projections from a limited angle of view is available. Image reconstruction from such data is complicated since the sparse projection data does not contain sufficient information to completely describe the 3-D body. Statistical (Bayesian) inversion is well-suited approach for reconstruction from such incomplete data. In statistical inversion, *a priori* information is used to compensate for the incomplete information of the data. In this talk, a statistical model for 3D x-ray tomography with sparse data is presented. Results are given with *in vitro* and *in vivo* projection data.

New developments in time-harmonic electromagnetic inverse obstacle scattering

Rainer Kress

University of Göttingen, Germany

For the approximate solution of the inverse obstacle scattering problem to reconstruct the boundary of an impenetrable obstacle from the knowledge of the far field pattern for the scattering of time-harmonic electromagnetic waves within the last decade a number of new reconstruction algorithms has been developed, analyzed and implemented. Roughly speaking one can distinguish three groups of methods. **Decomposition methods**, in principle, separate the inverse problem into an ill-posed linear problem to reconstruct the scattered wave from its far field pattern and the subsequent determination of the boundary of the scatterer from the boundary condition. **Iteration methods** interpret the inverse obstacle scattering problem as a nonlinear ill-posed operator equation and apply iterative schemes such as regularized Newton methods or Landweber methods for its solution. Finally, the third group consists of the more recently developed **sampling and probe methods**. In principle, these methods are based on criteria in terms of the solvability of ill-posed linear integral equations of the first kind to decide whether a point lies inside or outside the scatterer. The talk will give a survey by describing one or two representatives of each group including a discussion on the various advantages and disadvantages.

Around the circular Radon transform

Peter Kuchment

Texas A&M University, USA

The (restricted) circular Radon transform integrates a function over the set of all spheres centered at a given set. Such transforms arise in a variety of topics from approximation theory, to complex analysis, to mathematical physics, to inverse problems, including for instance some newly developing types of tomography. Some of the standard problems of uniqueness of reconstruction of a function from such data, inversion formulas, and range of this transform seem to be hard and have not been completely resolved. The talk will survey some known and provide some new results concerning this problem.

A first step towards nonlinear online parameter estimation

Philipp Kügler

University of Linz, Austria

Given a dynamic system $\dot{y} = f(q_*, y, t)$ in a (possibly infinite dimensional) Hilbert space Y, we consider the problem of estimating the unknown parameter q_*, element of another (possibily infinite dimensional) Hilbert space X, simultaneously to the evolution of the physical state y from observations of the latter. Even if f is linear in y, this inverse problem typically involves a nonlinear parameter-to-output-map. So far, online estimation techniques were only discussed in the context of linear and finite deminensional problems. We present an online parameter estimator for nonlinear problems based on the minimization of a weighted total prediction error and utilize Lyapunov theory for its analysis. Numerical results are presented.

Anisotropic inverse problems: uniqueness and stability

Yaroslav Kurylev

Loughborough University, UK

This is a joint work with M.Anderson, A.Katsuda, M.Lassas and M.Taylor. It is devoted to finding conditions on a Riemannian maniflod to establiosh (conditional) stability in the inverse boundary spectral problem and inverse boundary problem for the heat equation on a manifold. These conditions are defined in terms of bounds for the curvature, second fundamental form, diameter and injectivity radii and are related to the conditions of precompactness (in the Gromov-Hausdorff topology) of certain class of manifolds with boundary and optimal regularity in these classes of manifolds.

Follytons and the removal of eigenvalues for fourth order differential operators

Ari Laptev

KTH, Sweden

A non-linear functional $Q[u, v]$ is given that governs the loss, respectively gain, of (doubly degenerate) eigenvalues of fourth order differential operators $L = \partial^4 + \partial\, u\, \partial + v$ on the line. Apart from factorizing L as $A^*A + E_0$, providing several explicit examples, and deriving various relations between u, v and eigenfunctions of L, we find u and v such that L is isospectral to the free operator $L_0 = \partial^4$ up to one (multiplicity 2) eigenvalue $E_0 < 0$. Not unexpectedly, this choice of u, v leads to exact solutions of the corresponding time-dependent PDE's.

Non-uniqueness in the parameter identification for anisotropic elastodynamics

Anna L. Mazzucato

Penn State University, USA

We study the problem of unique identification of smoothly varying material parameters for anisotropic hyperelastic media by dynamic displacement-to-traction measurements made at the boundary. We show that the dynamic Dirichlet-to-Neumann map may determine the material parameters only up to diffeomorphisms that fix the boundary. We adapt the covariant formulation of the system of elastodynamics as discussed by Marsden and Hughes from Eulerian to Lagrangian coordinates, so that the underlying balance laws are respected. We then study the orbits of elasticity tensors under the action by pullback via diffeomorphisms. We give a partial, point-wise characterization of such orbits employing a canonical representation of fourth-order symmetric tensors in term of harmonic and Cartan decompositions. For example, uniqueness in the parameter identification for isotropic materials gives rise to partial uniqueness for certain anisotropic (possibly composite) media. This is joint work with Lizabeth Rachele.

Optical tomography on Riemannian surfaces

Stephen McDowall

Western Washington University, USA

Optical tomography refers to the use of near-infrared light to determine the optical absorption and scattering properties of a medium. In the stationary Euclidean case the dynamics are modeled by the radiative transport equation which assumes that, in the absence of interaction, particles follow straight lines. Here we shall study the problem in the presence of a Riemannian metric where particles follow the geodesic flow of the metric. In particular we study the problem in dimension two where the analysis is more delicate than in the higher dimensional cases.

Scattering and complete metrics on manifolds with corners

Richard Melrose

MIT, USA

In this talk I will discuss geometric generalizations of scattering theory in which Euclidean space is replaced by the interior of a compact manifold with boundary or corners. In particular I will concentrate on classes of complete metrics on the interior of such a manifold which leads to 'asymtotically commutative' behaviour analogous to Euclidean space deformed by a large (but bounded) potential.

The D-bar method in electrical impedance tomography

Jennifer Mueller

Colorado State University, USA

The D-bar method of inverse scattering has been shown to give a constructive method for solving the inverse conductivity problem posed by Calderon in 1980. This talk gives an overview of the D-bar method and the associated numerical algorithm for 2-D EIT. Reconstructions are presented from phantom and human chest data.

The inverse spectral theory of the periodic Euler-Bernoulli equation

Vassilis Papanicolau

National Technical University of Athens, Greece

We discuss the general inverse spectral theory of the periodic Euler-Bernoulli equation. The theory is an analog of the Hill theory. The key ingredient is Abel's Theorem for BAker-Akhiezer functions.

A systematic view on the basic setup and frequent questions for inverse problems

Roland Potthast

University of Göttingen, Germany

The area of inverse problems for partial differential equations is concerned with the reconstruction of shapes or material properties from measurements of fields in some distance to the location of the desired quantities. We will propose some multidimensional framework for the systematic investigation of the questions related to the solution of inverse problems.

The world of inverse problems is represented as a set of "universes" with different "galaxies" as components. Each galaxy consists of a set of metric or topoligical spaces which in detail describe the realization of the particular setting, a set of logical forms describing typical statements like "convergence" or further structures.

Our "world" includes what we call the "setting universe" (geometric setup, PDE, boundary condition, excitation, measurements and relevant features), the "analysis universe" (uniqueness, existence, stability, regularity, asymptotics), the "algorithms universe" (recipe, convergence, stability, complexity, efficiency, implementation) and the "application universe" (problem clouds, education, institution, time, manpower, goal cloud, ethics, usefullness, profit).

We will present the systematic view and discuss its usefullness and application.

On Calderón's inverse conductivity problem and quasiconformal maps in two dimensions

Lassi Päivärinta

University of Helsinki, Finland

We show that the Dirichlet to Neumann map for the equation $\nabla \cdot \sigma \nabla u = 0$ in a two dimensional domain uniquely determines the bounded measurable conductivity σ. This gives a positive answer to a question of A. P. Calderón from 1980. Earlier the result has been shown only for conductivities that are sufficiently smooth. In the proof we use the connection between the conductivity equation and quasiconformal maps and apply the well-established theory of these maps at several points of the talk. At the end of the talk we discuss the corresponding result for anisotropic conductivities.

This is a joint study with Kari Astala and in the anisotropic case with Astala and Matti Lassas.

Inverse problems for anisotropic elastic media

Lizabeth Rachele

University at Albany, SUNY, USA

We consider the dynamic parameter identification problem for bounded, three-dimensional isotropic and anisotropic elastic media with smoothly varying density and elastic properties. Displacement-to-traction surface data for the inverse problem is modeled by the Dirichlet-to-Neumann map on a finite time interval.

In this talk I will discuss the inverse problem for those anisotropic elastic media in which disturbances propagate along geodesics with respect to Riemannian metrics. We say that such media have *geodesic wave propagation* (GWP). Isotropic elastic media have GWP, as do (isotropic) elastic media with residual stress. Here we introduce two 5-parameter families $\{(C, \rho)\}$ of transversely isotropic elastic media with geodesic wave propagation.

This is joint work with Anna Mazzucato.

A variational approach to the reconstruction of potentials and cracks by boundary measurements

Luca Rondi

Università degli Studi di Trieste, Italy

We present a novel approach to the reconstruction issue for the inverse crack problem, that is for the determination of fractures, and other defects as well, inside a conductor by electrostatic boundary measurements.

We observe that the cracks, or the boundaries of other defects such as cavities or material losses at the boundary, contain and consist of the jump sets of the electrostatic potentials inside the conductor. Therefore our aim is to recover the potential given the Cauchy data (the prescribed current density and the measured potential) on a known and accessible part of the boundary.

We prove that the potential is uniquely determined, even if the defects are unknown, by its Cauchy data. Then we show that the potential is the unique minimum of a functional which depends on the Cauchy data only. Such a minimum problem is set inside the space of special functions of bounded variation and the functional to be minimized is based on the so-called Mumford-Shah functional.

The Mumford-Shah functional is not easy to handle from a numerical point of view. Furthermore, we have to take into account the fact that the Cauchy data are noisy and that the problem is severely ill-posed, hence some regularization is needed. We deal with these three issues at once by constructing a sequence of functionals defined on smooth functions whose minima converge to the potential to be recovered.

This method provides a reconstruction procedure which does not require any a priori information either on the number or on the topology of the defects (we may treat cracks, cavities and material losses at the boundary simultaneously) and which never requires to solve the direct problem.

Singular and quadratic eigenvalue problems

William Rundell

Texas A&M University, USA

Scattering and inverse scattering on asymptotically hyperbolic manifolds

Antônio Sá Barreto

Purdue University, USA

We define radiation fields on asymptotically hyperbolic manifolds and use them to obtain the scattering matrix. We also use the radiation fields to prove that the scattering matrix at all energies determines the manifold and the metric up to isometries.

On inverse scattering from random potentials

Eero Saksman

University of Jyväskylä, Finland

The talk describes a joint work with Matti Lassas and Lassi Päivärinta, University of Helsinki. Let q be a compactly supported (random) potential on the plane $\mathbf{R}^2$. Assume that $x_0 \in \mathbf{R}^2$ lies outside the support of q. We consider inverse Schrödinger back-scattering from the potential q with incident fields corresponding to point sources nearby x_0.

The class of potentials q considered includes locally isotropic non-smooth Markov fields. By letting C_q stand for the covariance operator of q, the principal part of the inverse operator $(C_q)^{-1}$ has the form $a\Delta$. The function a can be thought to describe the material parameters.

Our main result implies that almost surely one can recover the coefficient a from energy averages of the measurements over different frequences, using only back-scattering data near the point x_0. Somewhat surprisingly, this can be be done by using measurements from a single realization of the potential q only. Our analysis treats the full non-linear problem without employing approximative assumptions.

Reconstruction of singularities in two dimensional Schrödinger operator with fixed energy

Valeri Serov

University of Oulu, Finland

We prove that in dimension two potential scattering the singularities of the unknown potential are obtained exactly by the Born approximation which corresponds to the scattering data with fixed energy. The proof is based on the new estimates for the Green-Faddeev's function in the weighted spaces $L^p_\sigma(R^2), 1 < p \leq 2$. These estimates allow us to consider the potentials with stronger singularities than in previous publications and with noncompact support.

Edge preserving regularization methods in image processing

Fiorella Sgallari

University of Bologna, Italy

Numerical inversion methods based on Faddeev Green's function

Samuli Siltanen

Gunma University / GE Healthcare, Finland

Exponentially growing solutions for the Schrödinger equation were introduced by Faddeev in 1966. Later, they became key tools in the solution of several inverse problems, such as electrical impedance tomography (EIT) and the Cauchy problem for the stationary Schrödinger equation. Recently, numerical construction of exponentially growing solutions in two dimensions has been implemented. Application of this construction to the following three areas is presented: (a) Inverse problem of electrocardiography, (b) Testing of the d-bar reconstruction method for EIT, (c) Construction of solutions to a (2+1)-dimensional KdV equation (with no smallness assumption on initial data) via the inverse scattering method.

Continued fractions and their continuous analogs in inverse spectral theory

Barry Simon

Caltech, USA

I'll discuss inverse theory for three spectral problems: orthogonal polynomials on real line and on the unit circle and half line Schrodinger operators. For OPRL, there is a continued fraction approach going back to Steiljes and for OPUC going back to Schur and Geronimus. After presenting that approach, discuss the analogous A-function solution of the inverse problem for the Schrodinger case.

An application of adjoint dispersion equation and data assimilation methods to inverse dispersion problem

Mikhail Sofiev

Finnish Meteorological Institute, Finland

An inverse dispersion problem in its general form requires finding an unknown source of atmospheric tracer from information of its distribution from monitoring sites, possibly located far away from the source place. As usual, a straightforward solution does not exist since the diffusive component of atmospheric transport is irreversible. However, with the adjoint dispersion technique this problem can still be approached. Current presentation makes an attempt to interpret the results of the adjoint dispersion calculations in two ways: as a sensitivity of the given set of monitoring stations to locations of the source; and as a basis for 4-dimensional variational data assimilation (4D-VAR) for the source intensity. Both interpretations can lead to an approximate solution of the inverse problem either in statistical or in deterministic form. The methodology is tested on real data from the European Tracer Experiment ETEX. It consisted of release of a passive tracer in Western France with the pollution cloud being followed by 150 ground-based monitoring stations spread around Europe. The concentrations in air were reported by these sites with 3-hour averaging during the subsequent 3 days since the release. This allowed following the tracer path over the whole of Western and Central Europe, with part of mass spotted in Scandinavia. Since the emission parameters were well known, this dataset is ideal for evaluation of various numerical techniques. Both direct and adjoint dispersion simulations were made with a Finnish emergency modeling framework SILAM. The core of the model consists of Lagrangian advection scheme with Monte-Carlo random-walk module simulating diffusion process. Input data are treated with the meteorological pre-processor allowing flexible setup of the data source and both forward and backward motion along the time axis. An application of SILAM computations together with the above treatment of the output have shown quite encouraging results and allowed to delineate the source location both in space and time.

Boundary rigidity of Riemannian manifolds

Plamen Stefanov

Purdue University, USA

Let (M, g) be a compact Riemannian manifold with boundary. We study the inverse problem of recovering g (up to an isometry) from the distance function $d(x, y)$ known for all boundary points. The linearized problem is recovery of a tensor (up to a potential field) from the geodesic X-ray transform. We show that there is local uniqueness and Hölder stability for generic simple metrics. To this end, we prove uniqueness and a stability estimate with loss of one derivative for the linear problem for an open dense set of simple metrics including all real analytic (simple) ones. The talk is based on a joint work with Gunther Uhlmann.

Deductions about size and location based on scattering data

John Sylvester

University of Washington, USA

There are many successful techniques for deducing the location of point sources or scatterers from a limited number of acoustic or electromagnetic measurements. These measurements are far too few to uniquely identify a general source or even give an upper bound on its support. Nevertheless, the task of remote sensing is to infer what we can about size and location from exactly such limited data sets.

In several cases, will show that this data does uniquely determine a lower bound on a suitably defined notion of support of a source or scatterer.

We will take the Helmholtz equation as a model and consider some specific data sets, i.e.

1) broadband (many frequencies) measurements at a few angles
2) a single frequency far field measured from multiple angles (i.e one monochromatic incident wave, many sensors)
3) single frequency (multi-angle) backscattering data

In the last two cases we can find a lower bound on the convex hull of the support and a similar but weaker notion in the first case.

We will discuss the spectrum of the operator which maps sources to far fields and describe the role it plays in the computation of what we wil call the convex scattering support of the data.

Reconstruction of the convection terms in two dimensional domains

Alexandru Tamasan

University of Toronto, Canada

I consider an inverse boundary value problem for a convection diffusion process. From the Dirichlet to Neumann map we can reconstruct the unknown convection field (assuming homogeneous diffusivity). One example is the impedance tomography in isotropic media. One can think of conductivity as a convection vector field. The main ingredient is inverse scattering for the D-bar equation. This can be seen as a diagonalization of Beals and Coifman's formalism.

Inverse problems for scalar conservation laws

Katsumi Tanuma

Gunma University, Japan

We consider an inverse problem for the scalar conservation law. The scalar conservation law is described by

$$u_t + f(u)_x = 0, \qquad t > 0, \quad x \in \boldsymbol{R}, \tag{1}$$

with the initial data

$$u(x, 0) = u_0(x), \qquad x \in \boldsymbol{R}. \tag{2}$$

We assume that the function f, called the flux function, is $C^2(\boldsymbol{R})$ and uniformly convex, i.e., $f''(u) \geq C$ for some $C > 0$. Then it is well known that for arbitrary bounded measurable initial data $u_0(x)$ there exists uniquely a global weak solution $u(x,t)$ to (1), (2) which satisfies the entropy condition. In general, $u(x,t)$ develops discontinuities (shocks) in a finite time even though $u_0(x)$ is smooth. If $u_0(x)$ is a non-negative continuous function with compact support, then several shocks are generated at several points in a finite time. They collide each other as the time proceeds and after a while finally merge to form a single shock [Whitham(1974), Tanuma (1992)]. While the behavior of the shocks around the shock generation points and shock collision points is complicated, that of the single shock which results after generations and collisions of shocks is relatively simple and seems easy to be observed in reality.

The inverse problem we consider here is to reconstruct the unknown flux function f from the observation of the shock for large time corresponding to a single given initial data u_0. We show that f can be uniquely determined on some interval from the observation of the shock. Moreover, the interval on which f is uniquely determined can be taken arbitrarily large if we choose an appropriate sequence of initial data. We then show that derivatives of f at a point can be reconstructed in an explicit way from the asymptotics of the shock. We also show that not only the derivatives of f but also those of the initial data u_0 can be reconstructed from the additional information on the shock amplitude.

One outstanding example modelled by a scalar conservation law concerns with the traffic flow on a highway [LeVeque (1990), Whitham]. Here the unknown $u(x,t)$ denotes the density of cars, the number of cars per unit length at the point x on the road and at the time t, and $f(u)$ denotes a flux of cars, the number of cars which cross each position per unit time. The latter will be determined by conditions such as the shape of the road, human reaction, etc. Then the shock $x(t)$, i.e., the point across which the density is discontinuous, can be regarded as a generation point of the traffic jam. The inverse problem in this case corresponds to the problem of determining the unknown flux function $f(u)$ along the highway from the information on the behavior of the location of the traffic jam. Thus the results of this paper may be used to design the flux function for a given situation of the road.

Travel time tomography and boundary rigidity

Gunther Uhlmann

University of Washington, USA

We discuss in this talk some recent results on the boundary rigidity problem which consists in determining a Riemannian metric of a compact Riemannian manifold with boundary by measuring the lengths of geodesics joining points of the boundary.

Geometric optics and the wave equation on manifolds with corners

András Vasy

MIT / Northwestern, USA

In this talk I will describe the propagation of smooth (C^∞) and Sobolev singularities for the wave equation on smooth manifolds with corners M equipped with a Riemannian metric g. That is, for $X = M \times \mathbb{R}_t$, $P = D_t^2 - \Delta_M$, and $u \in H^1_{\mathrm{loc}}(X)$ solving $Pu = 0$ with homogeneous Dirichlet or Neumann boundary conditions, the appropriate wave front set $\mathrm{WF}_b(u)$ of u is a union of maximally extended generalized broken bicharacteristics. Since the latter follow the rules of geometric optics, i.e. those of classical dynamics, this result reflects yet another facet of the classical-quantum correspondence, namely that *singularities* of solutions of the wave equation follow geometric optics. This result is a smooth counterpart of Lebeau's results for the propagation of analytic singularities on real analytic manifolds with appropriately stratified boundary.

I will indicate the key ideas of the proof, such as microlocalization with respect to the appropriate pseudodifferential algebra, $\Psi_b(X)$ (boundary or b-microlocalization), and gaining b-regularity (i.e. conormal regularity) relative to $H^1_{\mathrm{loc}}(X)$ via positive commutator estimates. Certain aspects of this problem are related to N-body scattering.

Borg-Marchenko two spectra uniqueness theorem for Schrödinger operators with continuous spectrum

Ricardo Weder

Universidad Nacional Autónoma de México, Mexico

In this talk I will discuss recent joint results with T. Aktosun on the Schrödinger equation on the half line with a real-valued, integrable potential having a finite first moment. It is shown that the potential and the boundary conditions are uniquely determined by the data containing the discrete eigenvalues for a boundary condition at the origin, the continuous part of the spectral measure for that boundary condition, and a subset of the discrete eigenvalues for a different boundary condition. This result extends the celebrated two-spectra uniqueness theorem of Borg and Marchenko to the case where there is also a continuous spectrum.

Furthermore, I will present recent joint results with M. Lassas and V. Kurylev in a multi-dimensional analogue of the Borg-Levinson two spectra theorem. We consider multi-dimensional Schrödinger operators defined in a compact set, with Robin boundary condition. We prove that the eigenvalues, and the Gateaux derivative of the eigenvalues with respect to the parameter in the Robin boundary condition, uniquely determine both the potential and the Robin parameter.

Stability in inverse problems by Carleman estimates

Masahiro Yamamoto

University of Tokyo, Japan

Inverse problem of reconstruction of velocity and density in a weakly lateral heterogeneous half-space

Victor Zalipaev

Loughborough University, UK

A wave propagation generated by a boundary source into a weakly lateral heterogeneous (WLH) half-space is considered. The acoustic wave equation, which desrcibes the process includes velocity and density. WLH medium means that both functions depend on horizontal coordinates myltiplied by epsilon, which is a small parameter, opposite to a strong dependense on depth z - coordinate. The problem of calculating velocity and density inside the half-space from knowledge of the input pulse as a boundary sourse and measured data at z=0 is the inverse problem in reflection seismology. The talk describes an approach in which a recurrent system of wave equations for representation of the solution to the corresponding inverse problem in the form of perturbation series with respect to epsilon is ontained. In the zero-order approximation for the velocity and density we have 1D inverse problem which may be reduced to a system of non-linear Volterra integral equations (A.S.! Blagovestchenskii algorithm) . Next order approximations are determined as solution of coupled linear Volterra system of integral equations. We demostrate the effectiveness of the aproach in the numerical analysis by considering 2D case of velocity reconstruction.