

ON REDUCED ARAKELOV DIVISORS OF A NUMBER FIELD

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ABSTRACT. Let F be a number field. The Arakelov class group Pic_F^0 of F is an analog to the Picard group of a curve. From this group one can read off the class number h_F and the regulator of F . A good tool to compute Pic_F^0 is the class of reduced Arakelov divisors of F . In the first part, we recall the definitions of Arakelov divisors and the Arakelov class group of a number field and its structure. In the second part, we first discuss “nice” properties of reduced Arakelov divisors. Then we generalize the concept of reduced Arakelov divisors and introduce C -reduced divisors for a given number $C \geq 1$ as well as their properties. Finally we show one of its applications: computing the function h^0 for number fields with unit groups of rank at most 2.

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