Spring school in harmonic analysis and PDE 2008.

Helsinki University of Technology, June 2-6, 2008.

All lectures are in the hall K in the main building.

Monday.

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10.15-11 Grafakos I
11.15-12 Duzaar I
14.15-15 Martell I
15.15-16 Nystrm
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Tuesday.

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10.15-11 Urbano I
11.15-12 Grafakos II
14.15-15 Mingione I
15.15-16 Zhong
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Wednesday.

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10.15-11 Grafakos III
11.15-12 Martell II
14.15-15 Duzaar II
15.15-16 Urbano II
16.15-17 Lindqvist
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Thursday.

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10.15-11 Mingione II
11.15-12 Duzaar III
14.15-15 Urbano III
15.15-16 Martell III
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Friday.

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10.15-11 Mingione III
12.15-13 R.T. Rockafellar
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Boundary regularity for non-linear elliptic and parabolic systems

Frank Duzaar, University of Erlangen-Nuremberg

We consider non-linear elliptic and parabolic systems of the type

$$-\text{div}\ a(x,u,Du) = 0 \qquad \text{in } \Omega,$$

respectively

$$u_t - \text{div}\ a(x,t,u,Du) = 0 \qquad \text{in } \Omega_T$$

with Hölder continuous dependence on (x,u) respectively (x,t,u), and give conditions guaranteeing that $\mathcal{H}^{n-1}$- respectively $\mathcal{H}^n$-almost every boundary point of $\partial\Omega$, respectively $\partial_{\mathrm{b}}\Omega_T$, is a regular point for the (spatial) gradient of solutions to related Dirichlet problems, respectively initial Cauchy-Dirichlet problems. Here Ω is a bounded domain in $\mathbb{R}^n$ and $T > 0$. In the parabolic case Ω_T stands for the parabolic cylinder $\Omega \times (-T, 0)$ and $\partial_{\mathrm{b}}\Omega_T$ denotes the lateral boundary $\partial\Omega \times (-T, 0)$.

The lectures are divided into three parts. In the first one we treat the partial regularity problem, i.e. we give conditions under which the solutions are regular, i.e. the (spatial) gradient is Hölder continuous. In the case of elliptic systems we will introduce the method of $\mathcal{A}$-harmonic approximation, while for parabolic systems we show how the parabolic analogue – the method of $\mathcal{A}$-caloric approximation – yields the corresponding regularity result. Once this has been achieved we show in the second part of the lectures how to obtain the existence of regular boundary points for solutions of Dirichlet-problems. In the third and last part we will explain recent results concerning the existence of regular boundary points for solutions to non-linear parabolic systems.

REFERENCES

[1] Bögelein V. & Duzaar F. & Mingione G.: Boundary regularity for nonlinear parabolic systems. *Preprint 2008.*

[2] Duzaar F. & Grotowski J.F.: Optimal interior partial regularity for nonlinear elliptic systems: the method of A-harmonic approximation. *manuscripta math. 103 (2000), 267-298.*

[3] Duzaar F. & Kristensen J. & Mingione G.: The existence of regular boundary points for non-linear elliptic systems. *Journal fr die reine und angewandte Mathematik (Crelles Journal) 602 (2007) 17-58.*

[4] Duzaar F. & Mingione G. & Steffen K.: Parabolic Systems with Polynomial Growth and Regularity. *Preprint 2008.*

[5] Grotowski J.F.: Boundary regularity for nonlinear elliptic systems. *Calc. Var. Partial Differential Equations 15 (2002), 353-388.*

[6] Hamburger C.: Partial boundary regularity of solutions of nonlinear superelliptic systems. *To appear*

[7] Kristensen J. & Mingione G.: The singular set of ω-minima. *Arch. Ration. Mech. Anal. 177 (2005), 93-114.*

[8] Kristensen J. & Mingione G.: The singular set of minima of integral functionals. *Archive for Rational Mechanics & Analysis 184 (2007) 341-369.*

[9] Mingione G.: The singular set of solutions to non-differentiable elliptic systems. *Archive for Rational Mechanics & Analysis 166 (2003), 287–301.*

[10] Mingione G.: Bounds for the singular set of solutions to non linear elliptic systems. *Calc. Var. Partial Differential Equations 18 (2003), 373–400.*

[11] Mingione G.: Regularity of minima: an invitation to the dark side of the calculus of variations. *Applications of Mathematics 51 (2006), 355-425.*

The Carleson-Hunt theorem

Loukas Grafakos (University of Missouri at Columbia, U.S.A.)

We present the key ideas and several details that arise in the proof of the Carleson-Hunt theorem. We discuss Feffermans' proof of the almost everywhere convergence of Fourier integrals of square integrable functions, Lacey and Thiele's improvement of this proof, extensions to L^p, results concerning spaces near L^1, as well as open questions in the subject.

WEIGHTED NORM INEQUALITIES AND RUBIO DE FRANCIA EXTRAPOLATION

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The extrapolation theorem of Rubio de Francia is one of the most surprising results in the study of weighted norm inequalities in harmonic analysis: if an operator T is bounded on $L^2(w)$ for all $w \in A_2$, then T is automatically bounded on $L^p(w)$ for any p, $1 < p < \infty$, and every $w \in A_p$. Recently, in joint work with David Cruz-Uribe and Carlos Pérez, we have obtained a more simplified proof that allows different extensions of this beautiful result. One can derive weighted estimates on function and modular spaces, and also vector-valued inequalities. Besides, we consider extrapolation results from the class of weights A_∞ motivated by the Coifman-Fefferman estimate, where a Calderón-Zygmund operator with smooth kernel is controlled by the Hardy-Littlewood maximal function in $L^p(w)$ for all $0 < p < \infty$ and every $w \in A_\infty$. We will give applications of extrapolation to the study of weighted norm inequalities.

Towards a non-linear Calderón-Zygmund theory

Giuseppe Mingione (University of Parma, Italy)

Classical Calderón-Zygmund theory deals with integrability properties of solutions to linear elliptic and parabolic equations. The considerable developmnet of non-linear technologies of the last years has led to establish regularity estimates for solutions to possible degenerate non-linear equiations, which, in turn, can be used to build a series of results actually converging to what we might call a non-linear Calderón-Zygmund theory. It is my purpose to discuss some of such aspects.

Boundary Harnack inequalities for operators of p-Laplace type with variable coefficients

Kaj Nyström

In this talk I will describe recent results (joint work with J. Lewis and N. Lundstrm) concerning boundary Harnack inequalities and the Martin boundary problem for operators of p-Laplace type with variable coefficients in Reifenberg flat domains.

Implicit Functions and Solution Mappings in Variational Analysis

R.T. Rockafellar (University of Washington, Seattle)

In traditional mathematics, problems could generally be posed as solving equations, and the question of how a solution might depend on parameters was answered by the classical implicit function theorem. Nowadays, the concept of a problem is much richer and can involve more than just equations. Problem models in terms of optimization, equilibrium, and variational inequalities are important in many applications, for instance. The same question arises of dependence of solutions on parameters, but finding the answers has been the subject of much research.

In 1980, S.M. Robinson published a powerful theorem about solutions to parameterized variational inequalities which, in particular, could represent the first-order optimality conditions in a nonlinear programming problem dependent on parameters. His result covered much of the classical implicit function theorem, if not quite all, but it went far beyond that in ideas and format. It was a landmark in demonstrating how questions of importance in optimization could breath new life into traditional topics in mathematics.

Advances in variational analysis now allow Robinson's theorem to be extended to "generalized equations" much broader than variational inequalities. His notion of first-order approximations can be utilized in the absence of differentiability in describing the effects of perturbations of the parameters. However, even looser forms of approximation are able to furnish significant information about the behavior of solutions.

The Method of Intrinsic Scaling: a systematic approach to regularity for degenerate and singular PDEs

José Miguel Urbano (CMUC-University of Coimbra, Portugal)

This set of lectures is an introduction to intrinsic scaling, a powerful method in the analysis of degenerate and singular PDEs.

The theory will be presented from scratch for the model case of the degenerate p-Laplace equation, bringing to light what is really essential in the method. An effort will be made to render the approach entirely self-contained, leaving aside technical refinements needed to deal with more general equations.

A review of potential applications of the theory to relevant models arising from chemotaxis, flows in porous media and phase transitions will also be addressed.

Quasilinear elliptic equations in the Heisenberg group

Xiao Zhong

University of Jyväskylä

Abstract

I will talk about the regularity of solutions to quasilinear elliptic equations of p-Laplacian type in the Heisenberg group. I will focus on the higher integrability of the horizontal and the vertical parts of the gradient.