Finnish Mathematical Days

Espoo 3.1. - 4.1.2008

Schedule

THURSDAY 3.1.2007:

	Event	Speaker	Location
9.00-9.45	*Registration*		Mellin Hall (Hall M)
09.45-10.00	Opening	Rector Matti Pursula	Mellin Hall (Hall M)
10.00-10.45	Plenary talk	Prof. Karl Sigmund	Mellin Hall (Hall M)
10.55-11.40	Plenary talk	Prof. Paavo Salminen	Mellin Hall (Hall M)
11.40-13.00	Lunch		Tuas Building
13.00- 14.30	Sessions 1-7	Various	H,J,N,L, U322, U345,U356
14.30-15.00	Coffee	Coffee	Coffee
15.00-16.00	Ph.D. Presentations	Various	Halls H,J and N
16.15-17.15	Sessions 1-7 (cont.)	Various	Various
19.00-22.30	Conference Dinner	Various	White Hall, Aleksanterinkatu 16-18

FRIDAY 4.1.2007:

	Event	Speaker	Location
09.00-09.10	Prizes of the Math. Union		Mellin Hall (Hall M)
09.10-09.55	Plenary talk	Prof. Stanislav Smirnov	Mellin Hall (Hall M)
09.55-10.25	Coffee	Coffee	Coffee
10.25-11.15	Plenary talk	Prof. Kaisa Nyberg	Mellin Hall (Hall M)
11.15-13.00	Lunch and posters		Alvari
13.00- 14.30	Sessions 1-7 Computer Demonstrations	Various	H,J,N,L, U322, U345,U356
14.30-15.00	Break		Free
15.00-16.30	Sessions 1-7 Computer Demonstrations	Various	H,J,N,L, U322, U345,U356
16.35	Closing the Days		L

Invited speaker (Hall M):

To 10.00-10.45	Prof. Karl Sigmund Between Freedom and Coercion: The emergence of costly punishment
To 10.55-11.40	Prof. Paavo Salminen Optimal stopping: applications, theory and history
Pe 9.10-9.55	Prof. Stanislav Smirnov Conformal invariance of the 2D Ising model
Pe 10.25-11.15	Prof. Kaisa Nyberg On linear Cryptanalysis

Ph.D. representations(Thu 15.00-16.00)

Sali H Chair: Tylli
Mika Leikas (Jyväskylän yliopisto): Incomplete projection families
Pekka Nieminen (Helsingin yliopisto): Composition Operators and Aleksandrov Measures

Sali J Chair: Högnäs
Jaakko Nevalainen (Tampereen yliopisto): Non-parametric Methods for Multivariate Location Problems with Independent and Cluster Correlated Observations
Jukka Lempa (Helsingin Kauppakorkeakoulu): On stochastic control of Markov processes

Sali L Chair: Päivärinta/Hella
Simo-Pekka Vänskä (Helsingin yliopisto): Beltrami-kenttien sirontaa
Meeri Kesälä (Helsingin yliopisto): Finitary abstract elementary classes

Sessions:

	H	**J**	**L**	**U322**	**U345**	**N**	**U356**
To 13.00-14.30	Tietok.opetus Rasila Apiola	Tilastotiede Oja	Analyysi I Martio	Kontrolli I Pohjolainen	Diskreetti I Kari	Teol. mat. Siltanen	Riskienhallinta Pennanen
To 16.15-17.15	Opetus K Näätänen	Stok. ja disk. Högnäs Oja	Analyysi K1 Latvala	Analyysi K2 Lindström	Diskreetti II Kari	Sov. mat. K Siltanen	Analyysi K3 Juutinen
Pe 13.00-14.30	Opetus I Näätänen	Stokastiikka I Högnäs	Analyysi II Latvala	Kontrolli II Malinen	Logiikka Hella	Sov. mat. Päivärinta	Numer. ja oper. Huhtanen
Pe 15.00-16.30	Opetus II Näätänen	Stokastiikka II Högnäs	Analyysi III Lindström	Mat.fysiikka Lukkarinen	Diskreetti K Kari	Analyysi K4 Alestalo	Mekaniikka Stenberg

K session perässä tarkoittaa kontribuoituja puheita.

Analyysi I (Martio) To 13-14.30 sali L

Riikka Korte (Teknillinen korkeakoulu): Hardy and fatness in metric spaces (30min)
Pekka Alestalo (Teknillinen korkeakoulu): Kuvausten isometrisestä approksimaatiosta (30min)
Lauri Ylinen (Helsingin yliopisto): Two-dimensional tomography with unknown view angles (30min)

Analyysi II (Lindström) Pe 13-14.30 sali L

Mahmoud Filali (Oulun yliopisto): Functions on a topological group (30min)
Jari Taskinen (Helsingin yliopisto): Toeplitz-operaattorit Bergman-avaruuksissa (30min)
Hans-Olav Tylli (Helsingin yliopisto): Composition operators on spaces of vector-valued functions (30min)

Analyysi III (Latvala) Pe 15-16.30 sali L

Peter Hästö (Oulun yliopisto): TBA (30min)
Petri Juutinen (Jyväskylän yliopisto): On a nonlinear mass transportation problem (30min)
Risto Korhonen (Joensuun yliopisto): Painleve type difference equations and finite-order meromorphic solutions (20min)

Analyysi K1 (Latvala) To 16.15-17.15 sali L

Glader Christer (Åbo Akademi): Nonlinear Riemann-Hilbert Problems with Circular Target Curves (20min)
Sirkka-Liisa Eriksson (Tampereen teknillinen yliopisto): Hyperbolic harmonic functions and their function theory (20min)
Timo Erkama (Joensuun yliopisto): Aritmeettiset jonot polynomien sykleissä (20min)

Analyysi K2 (Lindström) To 16.15-17.15 sali U322

Tuomas Hytönen (Helsingin yliopisto): Some new Carleson measure conditions and their connection to BMO (20min)
Ville Turunen (Teknillinen korkeakoulu): Pseudo-Differential Operators and Symmetries
Tero Vedenjuoksu (Oulun yliopisto): The Stone-Cech compactification of topological groups

Analyysi K3 (Juutinen) To 16.15-17.15 sali U356

Ville Suomala (Jyväskylän yliopisto): Which measures are projections of purely unrectifiable one-dimensional Hausdorff measures (20min)
Eeva Suvitie (Turun yliopisto): On a short sum involving an inner product of a holomorphic cusp form and a Maass form (20min)
Kevin Wildrik (Jyväskylän yliopisto): Doubling measures on incomplete spaces (20min)

Analyysi K4 (Alestalo) Pe 15-16.30 sali N

Heikki Orelma (Tampereen teknillinen yliopisto): Hyperbolinen funktioteoria geometrisissa algebroissa (20min)
Anne Maria Ernvall-Hytönen (Turun yliopisto): Elliptic Curves, Modular Forms... What's so fancy about them? (20min)
Vasudevarao Allu (Indian Institute of Technology Madras): Region of variability for certain classes of univalent functions satisfying differential equations (20min)

Diskreetti matematiikka I (Kari) To 13-14.30 sali U345

Erkko Lehtonen (Tampereen teknillinen yliopisto): Operations on finite sets, functional composition and ordered sets (30min)
Ion Petre (Åbo Akademi): The mathematics of gene assembly in ciliates (30min)
Alexander Okhotin (Turun yliopisto): Equations over sets of natural numbers (30min)

Diskreetti matematiikka II (Kari) To 16.15-17.15 sali U345

Vesa Halava (Turun yliopisto): Hilbert's 10th problem and undecidability in matrix equations (30min)

Diskreetti matematiikka K (Högnäs) Pe 15-16.30 sali U345

Esko Turunen (Tampereen teknillinen yliopisto): Axiomatic Extensions of Monoidal Logic (20min)

Pauliina Ilmonen (Tampereen yliopisto): On eigenvalues of meet and join matrices associated with incidence functions (20min)
Johanna Rämö (University of London): Äärelliset yksinkertaiset ryhmät ja kahden involuution tulot (20min)
Pirita Paajanen (University of Southampton): Aliryhmien kasvu ja ryhmien Zeeta-funktiot (20min)

Kontrolliteoria I (Pohjolainen) To 13-14.30 sali U322

Seppo Pohjolainen (Tampereen teknillinen yliopisto): Robustisuus ja säätö (30min)
Timo Hämäläinen (Tampereen teknillinen yliopisto): Robusti regulointi ääretönulotteisilla signaalingeneraattoreilla (30min)
Lassi Paunonen (Tampereen teknillinen yliopisto): Robust Stability of Observers (30min)

Kontrolliteoria II (Malinen) Pe 13-14.30 sali U322

Ville Havu (Teknillinen korkeakoulu): Cayley Transform as numerical integration scheme (30min)
Jarmo Malinen (Teknillinen korkeakoulu): Semigroups of impedance conservative boundary control systems (30min)
Pertti Palo (Teknillinen korkeakoulu): Recording speech during MRI (30min)

Logiikka (Hella) Pe 13-14.30 sali U345

Jonathan Kirby (University of Oxford): Model Theory of Holomorphic Functions (30min)
Keijo Heljanko (Teknillinen korkeakoulu): Verification Methods based on Propositional Logic Satisfiability (30min)
Juha Kontinen (Helsingin yliopisto): Äärellisten mallien teoria (30min)

Matemaattinen fysiikka (Lukkarinen) Pe 15-16.30 sali U322

Kalle Kytölä (Universit Paris-Sud): Punished loop-erased random walks (30min)
Mikko Stenlund (Courant Institute): Billiard and statistical properties of toral automorphisms (30min)
Jani Lukkarinen (Helsingin yliopisto): Effective dynamics of lattice systems in a kinetics scaling limit (30min)

Mekaniikka (Stenberg) Pe 15-16.30 sali U356

Teijo Arponen (Teknillinen korkeakoulu): Numerical and symbolic computation in multibody systems
Mika Malinen (CSC): On Preconditioning of Dissipative Acoustic Equations
Jari Mäkinen (Tampere University of Technology): TBA

Numeriikka ja operaattoriteoria (Huhtanen) Pe 13-14.30 sali U356

Marko Huhtanen (Teknillinen korkeakoulu): Matrix factorization problems (30min)
Santtu Ruotsalainen (Teknillinen korkeakoulu): Operator theory in diffractive optics (30min)
Jaroslav Zemanek (Mathematical Institute of the Polish Academy of Sciences): On powers and resolvents of linear operators (30min)

Opetus I (Näätänen) Pe 13-14.30 sali H

Tiina Rintala: Toiminnallista matematiikkaa alakoulujen Summamutikka-kerhoissa (30min)
Ville Tilvis: Maunulan matematiikkalukion toiminnasta (30min)
Matti Lehtinen ja Anne-Maria Ernvall-Hytönen: Amatööreistä huippuja? Matematiikan kilpailuvalmennus Suomessa (30min)

Opetus II (Näätänen) Pe 15-16.30 sali H

Juha Oikkonen (Helsingin yliopisto): Ajatuksia ja kokemuksia luentokursseista (30min)
Kyösti Tarvainen (Stadia): Ammattikorkeakoulujen insinööriopiskelijoiden matematiikan

osaamistaso ja sen parantaminen.(30min)
Juha Kinnunen (TKK): Näkökulma matematiikan ylioppilaskokeeseen (30min)

Opetus K (Näätänen) To 16.15-17.15 sali H

Timo Tossavainen (Joensuun yliopisto): Matematiikan opettajankoulutuksen haasteita ja mahdollisuuksia (20min)
Martti E. Pesonen (Joensuun Yliopisto) Dynaaminen Geometria (20min)

Riskienhallinta (Pennanen) To 13-14.30 sali U356

Ahti Salo (Teknillinen korkeakoulu): Diskreetin portfoliomallinnuksen sovelluksia riskienhallinnassa (30min)
Matti Koivu (Rahoitustarkastus): A stochastic model for assets and liabilities of a pension institution(30min)
Teemu Pennanen (Helsingin kauppakorkeakoulu): Pricing and hedging of claim processes in illiquid markets (30min)

Sovellettu matematiikka (Päivärinta) Pe 13-14.30 sali N

Jari Kaipio (Kuopion yliopisto): TBA (30min)
Tapio Helin (Teknillinen korkeakoulu): Hierarchical statistical inverse problems and the Mumford-Shah functional (30min)
Mikko Salo (Helsingin yliopisto): Carleman estimates and anisotropic inverse problems (30min)

Sovellettu ja teollisuusmatematiikka K (Siltanen) To 16.15-17.15 sali N

Stefan Geritz (Helsingin yliopisto): Biomathematics (20min)
Ping Yan (Helsingin yliopisto): Three-dimensional Competitive and Competitor-competitor-mutualist Lotka-Volterra Systems (20min)
Ahti Rahikainen (Teknillinen korkeakoulu): Biomechanics in shot put (20 min)

Stokastiikka I (Högnäs) Pe 13-14.30 sali J

Timo Koski (Kungliga tekniska hgskolan): TBA (30min)
Lasse Leskelä (Teknillinen korkeakoulu): Stochastic binary relations of random variables and processes (30min)

Stokastiikka II (Högnäs) Pe 15-16.30 sali J

Eero Saksman (Helsingin yliopisto): Fraktionaalisen Brownin liikkeen lokaali riippumattomuus (30min)
Tommi Sottinen (Reykjavik University): Local continuity (for stopping times) (30min)
Ilkka Norros (VTT): TBA (30min)

Stokastiikka ja Diskreetti matematiikka K (Högnäs/Oja) To 16.15-17.15 sali J

Christe Geiss (Jyväskylän yliopisto): The discrete-time hedging error for the Levy process model (30min)
Eija Laukkarinen (Jyväskylän yliopisto): Malliavin calculus for Levy processes and fractional smoothness (30min)

Teollisuusmatematiikka (Siltanen) To 13-14.30 sali N

Lassi Roininen (Sodankylän geofysikaalinen observatorio): Magnetometridatan aikasarja-analyysi tilastollisilla inversiomenetelmillä (30min)

Samuli Siltanen (Tampereen teknillinen yliopisto): Kolmiulotteinen röntgenkuvaus (30min)
Niilo Sirola (Tampereen teknillinen yliopisto): Henkilökohtainen navigointi (30min)

Tietokonavusteinen opetus (Rasila/Apiola) To 13-14.30 sali H

Matti Pauna (Helsingin yliopisto): WebALT virtuaalinen analyysin peruskurssi (30min)
Miika Huikkola (Tampereen teknillinen yliopisto): Opiskelijoiden asenteet ja taidot sekä tietotekniikan käyttö matematiikan opetuksessa (30min)
Antti Rasila (Teknillinen korkeakoulu): Kokemuksia harjoitustehtävien automaattisesta tarkastuksesta (30min)

Tilastotiede (Oja) To 13-14.30 sali J

Jukka Corander (Åbo Akademi): Bayesian unsupervised classification utilizing non-reversible parallel MCMC (30min)
Jarkko Isotalo (Tampereen yliopisto): A useful matrix decomposition and it's statistical applications in linear regression (30min)
Juha Karvanen (Kansanterveyslaitos): Use of L-moments in data description and modeling (30min)

Kuvausten isometrinen approksimointi
Pekka Alestalo

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Olkoon $A \subset \mathbf{R}^n$ ja $f\colon A \to \mathbf{R}^n$ funktio, jolla on seuraava ominaisuus: on olemassa sellainen $\varepsilon > 0$, että

$$|x-y| - \varepsilon \le |f(x) - f(y)| \le |x-y| + \varepsilon$$

kaikilla $x, y \in A$. Tarkastelemme kysymystä siitä, kuinka hyvin tällaisia funktiota voidaan approksimoida isometrioilla ja miten approksimoinnin tarkkuus riippuu joukon A geometrisista ominaisuuksista ja dimensiosta n. Lähtökohtana on Hyersin ja Ulamin tapausta $A = \mathbf{R}^n$ koskeva tulos ja sen viimeaikaiset yleistykset, joita olen tutkinut yhdessä D.A. Trotsenkon ja J. Väisälän kanssa.

BAYESIAN UNSUPERVISED CLASSIFICATION UTILIZING NON-REVERSIBLE PARALLEL MCMC

Jukka Corander

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Model-based classification is a useful tool for pattern exploration in a wide range of applications, ranging from pure data mining to allocation of observations to pre-defined classes. An unsupervised classification of observed data may be understood as a scientific learning task where a putative hidden structure underlying the observations is to be discovered. Although heuristic numerical methods for classification have been widely exploited for decades already, there has been a gradually increasing interest in the model-based approach as the computational restrictions have become less significant. Here we consider the general case of classifying items represented by discrete valued feature vectors. A predictive Bayesian classification approach is obtained by utilizing generalizations of the de Finetti type representation results under a model which generates stochastic partitions of data based on an random urn allocation scheme. Such a framework was formally derived in [1] in a molecular biological context. In particular, the stochastic partition framework enables a novel parallel search strategy based on non-reversible Markov chains, where distinct search processes can learn from each other in an unsupervised manner [2]. This classification strategy resolves the class identifiability problems associated with the latent class methods, and we illustrate that it can produce stable and sensible inferences in a situation where we observed a complete failure by the previously proposed MCMC algorithms.

This work has involved collaboration with Mats Gyllenberg and Timo Koski.

References:

[1] Corander, J., Gyllenberg, M. and Koski, T. (2007). Random Partition models and Exchangeability for Bayesian Identification of Population Structure. *Bull. Math. Biol.*, **69**, 797-815.

[2] Corander, J., Gyllenberg, M. and Koski, T. (2006). Bayesian model learning based on a parallel MCMC strategy. *Statist. Comput.* **16**, 355-362.

HYPERBOLIC HARMONIC FUNCTIONS AND THEIR FUNCTION THEORY

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There is a rich interplay between harmonic functions in the plane and holomorphic functions in the complex plane. In higher dimensions potential theory is very well developed but extensions of one variable complex function theory has many open problems. A higher dimensions a counterpart of complex numbers is a Clifford algebra which is a non commutative associative algebra that inherits the algebraic, geometric and metric properties of the Euclidean space $\mathbb{R}^n$. It is a generalization of the algebra of quaternions introduced by Hamilton around 1843. In 1935 Fueter defined regular functions in the algebra of quaternions and was able to prove the main integral theorems. Delanghe in 1970 was able to prove similar results in $\mathbb{R}^n$ using Clifford algebras.

Leutwiler noticed around 1990 that the power function, calculated using Clifford algebra, satisfy generalized Cauchy Riemann equations with respect to a hyperbolic upper half space model. Another important, but not so well known, fact is hyperbolic distance is Möbius invariant without any conformal factor. We study an extension of solutions of Leutwiler, called hypermonogenic functions and hyperbolic harmonic functions. We present new Cauchy type integral formulas and related results.

References:

1. Eriksson, S.-L., Integral formulas for hypermonogenic functions, *Bull. Bel. Math. Soc.* **11** (2004), 705–717.

2. Eriksson-Bique, S.-L. and Leutwiler, H., Hypermonogenic functions. In *Clifford Algebras and their Applications in Mathematical Physics*, Vol. 2, Birkhäuser, Boston, 2000, 287–302.

3. Eriksson, S.-L. (editor), *Clifford Analysis and Applications,* Tampere Univ. of Tech. Inst. of Math. Research Reports No 82 (2006).1–183.

4. Eriksson, S.-L., H. Leutwiler, *Hyperbolic Function Theory,* Adv. appl. Clifford alg. 17 (2007), 437–450.

ARITMEETTISET JONOT POLYNOMIEN SYKLEISSÄ

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Äärellinen n:n kompleksiluvun joukko S on polynomin P *n-sykli*, jos P:n rajoittuma joukkoon S permutoi S:n alkiot syklisesti. Jokaisella toisen asteen polynomilla $P \in \mathbf{C}[x]$ on tällaisia syklejä mielivaltaisen suurilla n:n arvoilla, mutta toistaiseksi vain arvoilla $n \leq 3$ on löydetty syklejä, joiden kaikki pisteet ovat **C**-*rationaalisia*, so. kuuluvat kuntaan $\mathbf{Q}(i)$. Otaksutaan, että pitempiä (toisen asteen polynomien) syklejä ei kunnassa $\mathbf{Q}(i)$ voi ollakaan, mutta tämä on toistaiseksi todistettu vain arvolla $n = 4$ [1].

Näissä sykleissä esiintyy silloin tällöin *aritmeettisia jonoja*, jos syklin pituus n on vähintään 3. Esimerkiksi polynomilla $P(x) = x^2+vx+v-1$ on aritmeettisen 3-jonon $\{1-v,\ 0,\ v-1\}$ sisältävä 4-sykli $\{1-v,\ 0,\ v-1,\ 2v(v-1)\}$, mikäli v on yhtälön $2v^3 - v^2 + 1 = 0$ juuri. Näytämme, että yleisesti toisen asteen polynomin **C**-rationaalinen n-sykli voi (pistejoukkona) sisältää aritmeettisen 3-jonon vain jos $n = 3$. Samalla tulee osoitetuksi, että Mandelbrotin joukon hyperbolisten komponenttien keskipisteet eivät kokonaislukuja 0 ja -1 lukuunottamatta ole **C**-rationaalisia.

References:

[1] T. Erkama: Periodic orbits of quadratic polynomials, Bulletin of the London Mathematical Society 38 (2006), 804-814.

[2] T. Erkama: Fermat'n jälkeen, Solmu 3/2006, 10-11.

[3] T. Erkama: Arithmetic progressions in cycles of quadratic polynomials, in preparation.

MODULAR FORMS, ELLIPTIC CURVES, AND WHY ARE THEY SO FANCY?

Anne-Maria Ernvall-Hytönen

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During recent years, the theory of modular forms and elliptic curves has become widely famous. The reasons for this are clear; the connections to Fermat's Last Theorem, and the applications to the elliptic curve cryptosystem are maybe the most important ones. However, the theory in itself is not so well-known.

During the talk, I will briefly go through the basics of the modular forms, and tell about their connections to elliptic curves. I will also explain why every positive integer can be represented as a sum of four squares, and tell some of the other highlights of the theory.

Finally, I will tell about my own research, which deals with the Fourier series of the holomorphic cusp forms. The results are in [1] and [2].

References:

[1] A.-M. Ernvall-Hytönen: On The Error Term in the Approximate Functional Equation for Exponential Sums Related to Cusp Forms, International Journal of Number Theory, to appear.

[2] A.-M. Ernvall-Hytönen, K. Karppinen: Upper bounds for exponential sums of Fourier coefficients of holomorphic cusp forms, International Mathematics Research Notices, to appear.

FUNCTIONS ON A TOPOLOGICAL GROUP

Mahmoud Filali

Abstract. I shall survey some of the recent results obtained jointly with colleagues on semigroup compactifications of a topological group G and on Banach algebras related to G. The common method with all these results is the construction at each time of an appropriate function (or operator when we are dealing with an algebra of operators) to deal with the problem.
Co-authors:Alaste, Bouziad, Monfared, Neufang, Protasov, Salmi, Vedenjuoksu.

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

The Discrete-Time Hedging Error for the Lévy Process Model

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Let (X_t) be a square integrable Lévy process with a characteristic triplet (γ, σ^2, ν) such that $S_t := e^{X_t}$ is a square integrable martingale. If one uses the exponential Lévy process (S_t) as price process, the market is not any more complete in general. That means for a payoff function f with $f(S_1) \in L_2$ the Galtchouk-Kunita-Watanabe decomposition implies

$$f(S_1) = \mathbb{E}f(S_1) + \int_0^1 \theta_t dS_t + N$$

where the random variable N is orthogonal to all stochastic integrals with respect to (S_t). In other words, N represents the residual risk of the payoff $f(S_1)$ which cannot be hedged. So the mean-variance hedging error is $\mathbb{E}(f(S_1) - \mathbb{E}f(S_1) - \int_0^1 \theta_t dS_t)^2 = \mathbb{E}N^2$.
If one takes into consideration that the portfolios are discretely adjusted in reality, one faces the additional error

$$\int_0^1 \theta_t dS_t - \sum_{k=1}^{n} \theta_{t_{k-1}}(S_{t_k} - S_{t_{k-1}}).$$

We investigate the relation between the convergence rate $r = \frac{1}{2}$ for specific chosen time nets $0 = t_0 < t_1 < \ldots < t_n = 1$ in

$$\left\| \int_0^1 \theta_t dS_t - \sum_{k=1}^{n} \theta_{t_{k-1}}(S_{t_k} - S_{t_{k-1}}) \right\|_{L_2} \leq cn^{-r}, \;\; n \to \infty,$$

and a certain fractional smoothness in Malliavin sense of the payoff function f.

References

[1] S. Geiss and M. Hujo: Interpolation and approximation in $L_2(\gamma)$. *Journal of Approximation Theory 144(2007)213-232.*

[2] E. Laukkarinen: Besov spaces, Lévy processes and Malliavin calculus. *preprint.*

[3] P. Tankov and E. Voltchkova: Asymptotic Analysis of Hedging Errors in Models with Jumps. *preprint.*

The effective dimension of the ecological feedback environment and the number of coexisting species

Stefan A. H. Geritz

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University of Helsinki

Abstract: Ecological feedback mechanisms prevent populations from becoming arbi- trarily large: e.g., food becomes scarce, predation rates and transmission rates of diseases increase as the population density becomes high, thus reducing the population growth rate. We assume that all interactions between individ- uals are channeled through environmental variables such as food availability, predation rates, infection rates, and so forth. The vector of all ecologically relevant environmental variables as a function of time we call the ecological feedback environment. Although the dimension of the space of all possible ecological feedback environments is potentially infinite, there often exists a finite-dimensional representation of this space. The smallest number of scalar variables neces- sary and sufficient to fully characterize the feedback environment as far as the population growth rate is concerned is called the effective dimension of the environment. The number of coexisting species generically cannot exceed the effective dimension of the ecological feedback environment. This is a very general and important result. But how does one in practise determine the effective dimension? I present a number of results that solve this question at least for ecologically similar species.s

NONLINEAR RIEMANN-HILBERT PROBLEMS WITH CIRCULAR TARGET CURVES

Christer Glader

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We consider the nonlinear Riemann-Hilbert problem with circular target curves: Given Hölder continuous functions c and r on the unit circle $\mathbb{T}$, with r strictly positive on $\mathbb{T}$, we seek functions w in the disk algebra $H^\infty \cap C$ of bounded holomorphic functions continuous on the closed unit disk, satisfying

$$|w(t) - c(t)| = r(t)\,, \quad \forall\, t \in \mathbb{T}\,.$$

This is also called the generalized modulus problem, see [3]. The complete set of solutions w can be described using an approach based on the interplay with the Nehari problem of best approximation by bounded holomorphic functions. The problems fall into three classes (regular, singular, and void), and emphasizing constructive aspects of the matter a Nevanlinna parametrization of the full solution set of regular problems can be derived.

When c and r^2 are rational functions all solutions of the problem are rational and can be constructed by solving interpolation problems of (generalized) Nevanlinna-Pick type. In the regular cases these interpolation problems are reduced to linear systems, which leads to efficient numerical methods.

The results have been obtained in collaboration with E. Wegert.

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HILBERT'S 10TH PROBLEM AND UNDECIDABILITY IN MATRIX EQUATIONS

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Let $\{A_1, A_2, \ldots, A_k\}$ be a finite set of $n \times n$ matrices with rational entries. It was proved by Babai et al. in [1] that if the matrices A_i are commutative, then it is decidable for any given $n \times n$ matrix M whether or not there exists natural numbers $j_1, j_2, \ldots, j_t$ such that:

$$A_1^{j_1} A_2^{j_2} \cdots A_k^{j_k} = M?$$

In other words, there exists an algorithm which answers "yes" if such numbers $j_1, \ldots, j_n$ exists which satisfy the equation, and answers "no" if no such indexes exist.

We shall examine a related problem where we consider the above equation for *non-commutative* integral matrices. We show that given the k matrices $A_1, A_2, \ldots, A_k \subseteq \mathbb{Z}^{n\times n}$, determining whether there exists natural numbers $i_1, i_2, \ldots, i_k$ such that:

$$A_1^{i_1} A_2^{i_2} \cdots A_k^{i_k} = Z,$$

where Z is the zero matrix, is undecidable. We do not use a reduction of Post's correspondence problem, as is standard for undecidability proofs in integer matrixes, we instead use the undecidability of Hilbert's tenth problem and properties of formal power series to show the undecidability. The reduction of the Hilbert's tenth problem to the equation of integer matrices is a variant of the reduction in [3].

The result is based on collaboration with P. Bell, T. Harju, J. Karhumäki and I. Potapov.

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CAYLEY TRANSFORM AS A NUMERICAL INTEGRATION SCHEME

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We interpret the Cayley transform of linear (finite- or infinite-dimensional) state space systems as a numerical integration scheme of Crank-Nicolson type. It has the following important property: if applied to a conservative (continuous time) linear system, then the resulting (discrete time) linear system is conservative in the discrete time sense. We show that for these systems the scheme is convergent from the input/output point of view.

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Hierarchical statistical inverse problems and Mumford-Shah functional

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In 1989 Mumford and Shah proposed their celebrated minimization method for image segmentation. The method combines shape and functional minimization thus making the computational problem challenging. In this talk we discuss the hierarchical statistical inverse problems and how the Bayesian methods can be applied to Mumford-Shah minimization.

This work has been done in collabration with Professor Matti Lassas, Helsinki University of Technology, Finland.

VERIFICATION METHODS BASED ON PROPOSITIONAL LOGIC SATISFIABILITY

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Computer aided verification is an approach that uses computerized algorithms to automatically search for design errors (bugs) in system designs such as complex microprocessors, safety critical control systems, and communication protocol implementations. In all these fields we build a mathematical model of the set of all behaviors the analyzed system has, and then check this model against the required design specifications expressed using e.g., linear temporal logic (LTL).

We report on a number of new techniques to make model checking more efficient for large systems. The approaches are based on mapping the temporal logic model checking problems into propositional logic satisfiability (SAT) problem [3, 2, 1], benefiting from recent advances in SAT solving technology.

The results have been done in collaboration with authors listed in the references below.

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MATRIX FACTORIZATION PROBLEMS

MARKO HUHTANEN *

Factoring matrices and operators into products of simpler ones is central and classical in matrix analysis [3, 4], numerical linear algebra [1] and operator theory [6, 2]. From the practical point of view, the LU and QR factorizations, as well as SVD are ubiquituous in everyday matrix computations.

Originally motivated by diffractive optics [5], in this talk we consider the problem of factoring a matrix $A \in \mathbb{C}^{n\times n}$ into the product of two matrices as

$$A = V_1 V_2$$

with the factors constrained to belong to prescribed subspaces $\mathcal{V}_1$ and $\mathcal{V}_2$ of $\mathbb{C}^{n\times n}$ over $\mathbb{C}$ (or $\mathbb{R}$).

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SOME NEW CARLESON MEASURE CONDITIONS AND THEIR RELATION TO BMO

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The inequality of John and Nirenberg tells that the means in the definition of bounded mean oscillation (BMO) can be equivalently taken in the sense of any L^p norm with a finite exponent p. On the other hand, the well-known characterizations of BMO in terms of Carleson measure conditions involve L^2 norms only.

In an effort to generalize some of these known results to the context of vector-valued functions, an L^p version of the Carleson measure condition emerged, which seems to be new even in the scalar-valued theory. As in the defining condition of BMO, one obtains a p-dependent scale of Carleson norms, all of which characterize the membership of a function in BMO.

In connection to some natural questions, such as the relation of BMO to the paraproduct operators (which are important in the $T(1)$ and $T(b)$ theorems), it may be more useful to employ the new L^1 Carleson condition than the classical L^2 version.

This is based in part on joint work with Lutz Weis (Universität Karlsruhe).

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ROBUST REGULATION OF INFINITE-DIMENSIONAL SYSTEMS WITH INFINITE-DIMENSIONAL EXOSYSTEMS

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One of the cornerstones of the classical automatic control theory of finite-dimensional linear systems is the Internal Model Principle (IMP) due to Francis and Wonham, and Davison. Roughly stated, this principle asserts that any error feedback controller which achieves closed loop stability also achieves robust (i.e. structurally stable) output regulation (i.e. asymptotic tracking/rejection of a class of exosystem-generated signals) if and only if the controller incorporates a suitably reduplicated model of the dynamic structure of the exogenous reference/disturbance signals which the controller is required to track/reject.

In this paper we discuss the state space generalization of the Internal Model Principle for infinite-dimensional systems and infinite-dimensional signal generators, which generate reference and disturbance signals of the form

$$\sum_{n=-\infty}^{\infty} a_n e^{i\omega_n t}, \qquad \omega_n \in \mathbb{R}, \quad (a_n)_{n\in\mathbb{Z}} \in \ell^1. \tag{1}$$

The presentation is based on the concept of the steady state behavior of the closed-loop system with inputs of the form (1). This approach leads us naturally to an infinite-dimensional Sylvester equation and a constrained infinite-dimensional Sylvester equation, which adds a constraint to the Sylvester equation for regulation. Then it is shown that feedback structure enables robustness, as the regulation equation is contained in the Sylvester equation and as the system reaches its steady state this equation is automatically satisfied. Finally it will be shown that if the controller contains a p-copy internal model of the exosystem, then the Sylvester equation implies robust regulation.

Due to the fact that the signal generator is infinite-dimensional, the closed-loop system cannot be exponentially stabilized. Instead strong stabilization must be used.

On Eigenvalues of Meet and Join Matrices Associated with Incidence Functions

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Abstract

Let $(P, \preceq, \wedge)$ be a locally finite meet semilattice. Let

$$S = \{x_1, x_2, \ldots, x_n\},\ x_i \preceq x_j \ \Rightarrow i \leq j,$$

be a finite subset of P and let f be a complex-valued function on P. Then the $n \times n$ matrix $(S)_f$, where

$$((S)_f)_{ij} = f(x_i \wedge x_j),$$

is called the meet matrix on S with respect to f. The join matrix on S with respect to f is defined dually. The number theoretic GCD and LCM matrices are the standard special cases of meet and join matrices.

In this paper we consider the eigenvalues of meet and join matrices with respect to f. There are no results published in the literature concerning the eigenvalues of meet and join matrices. We give a lower bound for the smallest eigenvalue of certain (real) positive definite meet and join matrices. We adopt an argument similar to that used by Hong and Loewy [1, Theorem 4.2] to power GCD matrices. Our lattice-theoretic approach, however, makes it possible to consider also LCM matrices with the same method (and matrices with respect to f). Further we give a region in which all the eigenvalues of a complex meet matrix $(S)_f$ with respect to f on meet closed set S and with respect to semimultiplicative f on join closed set S lie. Dually we give a region in which all the eigenvalues of a complex join matrix $[S]_f$ with respect to f on join closed set S and with respect to semimultiplicative f on meet closed set S lie. These results on complex meet and join matrices are new even for GCD and LCM matrices.

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A USEFUL MATRIX DECOMPOSITION AND IT'S STATISTICAL APPLICATIONS IN LINEAR REGRESSION

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It is well known that if $\mathbf{V}$ is a symmetric positive definite $n \times n$ matrix, and $(\mathbf{X} : \mathbf{Z})$ is a partitioned orthogonal $n \times n$ matrix, then

$$(\mathbf{X}'\mathbf{V}^{-1}\mathbf{X})^{-1} = \mathbf{X}'\mathbf{V}\mathbf{X} - \mathbf{X}'\mathbf{V}\mathbf{Z}(\mathbf{Z}'\mathbf{V}\mathbf{Z})^{-1}\mathbf{Z}'\mathbf{V}\mathbf{X}. \tag{*}$$

We show how useful we have found the formula $(*)$, and in particular its version

$$\mathbf{Z}(\mathbf{Z}'\mathbf{V}\mathbf{Z})^{-1}\mathbf{Z}' = \mathbf{V}^{-1} - \mathbf{V}^{-1}\mathbf{X}(\mathbf{X}'\mathbf{V}^{-1}\mathbf{X})^{-1}\mathbf{X}'\mathbf{V}^{-1} := \dot{\mathbf{M}}, \tag{**}$$

and present several related formulas, as well as some generalized versions. We also include several statistical applications.

This is a joint research with Simo Puntanen (University of Tampere, Finland) and George P. H. Styan (McGill University, Montreal, Canada).

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A NONLINEAR MASS TRANSPORTATION PROBLEM

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We consider the nonlinear optimal transportation problem of minimizing the cost functional $\mathcal{C}_\infty(\lambda) = \lambda\text{-}\operatorname{ess\,sup}_{(x,y)\in\Omega^2}|y-x|$ in the set of probability measures on Ω^2 having prescribed marginals. This corresponds to the question of characterizing the measures that realize the infinite Wasserstein distance. We establish the existence of "local" solutions and characterize this class with the aid of an adequate version of cyclical monotonicity. Moreover, under natural assumptions, we show that local solutions are induced by transport maps.

The results have been obtained in collaboration with Thierry Champion (Toulon) and Luigi de Pascale (Pisa).

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USE OF L-MOMENTS IN DATA DESCRIPTION AND MODELING

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L-moments [1], defined as expectations of linear combinations of order statistics, are alternatives for the central moments. L-moments may be used to describe the data and to estimate parametric distributions by equating the theoretical and the sample L-moments (method of L-moments). L-moment estimators are sometimes more efficient than maximum likelihood estimators.

The L-moment of order r can be defined as

$$L_r = \int_0^1 Q(u) P^*_{r-1}(u) \mathrm{d}u,$$

where $P^*_{r-1}(u)$ is the shifted Legendre polynomial of order $r-1$ and $Q(u) = F^{-1}(u)$ is the quantile function, i.e. the inverse of cumulative distribution function (cdf). All L-moments of a real-valued random variable exists if and only if the random variable has a finite mean and furthermore, a distribution whose mean exists, is uniquely determined by its L-moments.

Similarly to the central moments, L_1 measures location and L_2 measures scale. The higher order L-moments are usually transformed to L-moment ratios $\tau_r = L_r/L_2 \quad r = 3, 4, \ldots$ L-skewness τ_3 is related to the asymmetry of the distribution and L-kurtosis τ_4 is related to the peakedness of the distribution. Differently from the central moment skewness and kurtosis, τ_3 and τ_4 are constrained by the conditions $-1 < \tau_3 < 1$ and $(5\tau_3^2 - 1)/4 \leq \tau_4 < 1$.

Method of L-moments works very well with quantile mixtures [2]. Quantile mixtures are parametric families of distributions defined as a linear combination of quantile functions

$$Q(u) = \sum_{i=1}^{m} a_i Q_i(u),$$

where $Q_i(u)$ is a quantile function and a_i is a model parameter.

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Model theory of holomorphic functions

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One of the most important examples in model theory is the theory ACF of algebraically closed fields. It is the centre of a strong connection between model theory and algebraic geometry, which has been very productive. But what about complex analytic geometry? I will explain how some holomorphic functions have a well-behaved model theory, and how there are connections with deep questions in analytic geometry and even number theory.

MAJORITY IN LOGIC AND COMPUTATION

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The topic of the talk is Descriptive Complexity Theory. Descriptive complexity theory is a branch of Finite Model Theory which in general is concerned with the study of various logics over finite structures. The underlying idea behind descriptive complexity theory is that the complexity of an algorithmic problem can be understood as the richness of a language needed to specify the problem. In 1974 Fagin gave a characterization of nondeterministic polynomial time as the set of properties expressible in existential second-order logic. Later Stockmeyer showed that full second-order logic describes exactly the languages in the polynomial hierarchy (PH). The logarithmic and linear analogues of PH are also important complexity classes and as well they have been characterized logically in terms of alternating universal and existential quantifiers.

In this talk we consider the so-called counting extensions of these complexity hierarchies. We show that the corresponding counting classes can be logically characterized in an analogous way in terms of (first-order and second-order) majority quantifiers. We also discuss alternative characterizations of these classes in terms of threshold circuits and bounded arithmetics.

This is joint work with Hannu Niemistö (University of Helsinki).

PAINLEVÉ TYPE DIFFERENCE EQUATIONS AND FINITE-ORDER MEROMORPHIC SOLUTIONS

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Let $w(z)$ be an admissible finite-order meromorphic solution of the second-order difference equation

$$w(z+1) + w(z-1) = R(z, w(z)) \qquad (\dagger)$$

where $R(z, w(z))$ is rational in $w(z)$ with coefficients that are meromorphic in z. Then either $w(z)$ satisfies a difference linear or Riccati equation or else equation (†) can be transformed to one of a list of canonical difference equations. This list consists of all known difference Painlevé equations of the form (†), together with their autonomous versions. This suggests that the existence of finite-order meromorphic solutions is a good detector of integrable difference equations [1].

The results have been done in collaboration with R. Halburd.

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HARDY AND FATNESS IN METRIC SPACES

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We discuss the equivalence of three different conditions that can be used to describe the size of the boundary of a set. We work in Q–regular metric spaces but the results are partially new also in the Euclidean setting.

More precisely, we show that Q–Hardy's inequality, the uniform Q–fatness of the complement and the uniform perfectness of the complement are equivalent in certain situations. By uniform Q–fatness, we mean a capacitary version of the uniform measure density condition. It is well known that uniform p–fatness implies p–Hardy's inequality for all p also in metric spaces. We present a transparent proof that Q–Hardy's inequality implies uniform perfectness and that uniform perfectness further implies $(Q-\varepsilon)$–fatness in Q–regular spaces. As a by–product, we also obtain an easy proof for the self–improvement of uniform p–fatness in the case $p = Q$.

The results are based on joint work with Nageswari Shanmugalingam.

Penalized loop-erased random walks

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Abstract: Loop erasure of a two-dimensional simple random walk produces a simple curve, whose scaling limit is a conformally invariant random curve called SLE_2, [1]. I will discuss the loop-erasures of random walks penalized by their number of steps. In the continuum limit the underlying walk becomes a Brownian motion penalized by its local time, whereas the continuum limit of the loop-erasure is expected to become a random curve related to SLE_2 but failing to be conformally invariant. I will give expressions for the correlation functions of the local time in terms of fermionic quantum field theory. In view of these expressions and general quantum field theory considerations I will propose a description of the continuum limit of the loop-erasure and discuss some related mathematical challenges.

The talk is based on research done in collaboration with Michel Bauer (Service de Physique Théorique, CEA Saclay) and Denis Bernard (Laboratoire de Physique Théorique, Ecole Normale Supérieure), [2].

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Amatööreistä huippuja? Matematiikan kilpailuvalmennus Suomessa.

Matti Lehtinen ja Anne-Maria Ernvall-Hytönen

Monesta muusta urheilusta poiketen matematiikkakilpailujen yleinen lähtökohta on ollut tarkoitus tukea hyödylliseksi mutta ei kovin suosituksi katsottua matematiikan opiskelua. Maailmassa järjestetään vuosittain satoja erilaisia kansallisia ja kansainvälisiä kouluikäisille ja kouluja käyville suunnattuja matematiikkakilpailuja. Kytkös matematiikan kouluopetukseen on määrittänyt matematiikkakilpailujen tehtävänlaadintaa. Kilpailut ja kilpailutehtävät ovat kuitenkin ryhtyneet elämään pitkälti omaa elämäänsä eivätkä ne ole reagoineet matematiikan opetuksen melko radikaaleihin muutoksiin. Nykyään Suomessa eivät loistavatkaan matematiikan koulutiedot anna juuri ollenkaan eväitä kilpailutehtävien ratkaisemiseen.

Urheiluun ja kilpailemiseen sisäänrakennettu pyrkimys menestyä ja toisaalta koulu- ja kilpailumatematiikan eriytyminen ovat johtaneet erilaisten kilpailuvalmennusjärjestelmien syntyyn. Suomi on lähes poikkeuksetta lähettänyt joukkueen vuotuisiin kansainvälisiin matematiikkaolympialaisiin vuodesta 1973 lähtien. 1990-luvun puoliväliin mennessä oli tullut selväksi, että vähäiseenkään menestykseen pääseminen ei ole mahdollista ilman pitkäjänteistä valmennusta. Suomen matemaattisen yhdistyksen valmennusjaosto on yhteistyössä Päivölän kansanopiston kanssa vähin voimin pitänyt yllä valmennusjärjestelmää, johon kuuluu etä- ja kontaktivalmennusta. Kilpailutulokset eivät aina ole olleet häikäiseviä, mutta valmennuksen kautta on saatettu melko suuri joukko nuoria tietoiseksi siitä matematiikasta, joka on jossain koulumatematiikan ulko- ja yläpuolella. Matematiikan olympiavalmennus täyttää osaltaan sitä aukkoa, joka erottaa kouluissa annettavan laskento-orientoituneen opetuksen ja oikean, laajassa mielessä todistamisen idean ympärille rakentuvan matematiikan.

OPERATIONS ON FINITE SETS, FUNCTIONAL COMPOSITION, AND ORDERED SETS

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The unifying theme of this work [5] is the notion of functional composition. This notion is extended to function classes in [1], where compositions of clones of Boolean functions are investigated. Decompositions of the clone of all Boolean functions into minimal clones are seen to correspond to normal form systems of Boolean functions, including CNF, DNF, and Zhegalkin polynomials. The new median normal form system is shown to provide more efficient representations than the other normal form systems mentioned.

The second part of this work is a study of $\mathcal{C}$-minors. For a fixed class $\mathcal{C}$ of operations on a nonempty set A, an operation $f\colon A^n \to A$ is said to be a $\mathcal{C}$-minor of $g\colon A^m \to A$, if $f = g(h_1, \dots, h_m)$ for some $h_1, \dots, h_m \in \mathcal{C}$. Order-theoretical properties of certain $\mathcal{C}$-minor relations are examined in [3]. From the study of $\mathcal{C}$-minors defined by the clone of all projections on A arises the question how the number of essential variables of an operation is affected when variables are identified, and some answers are found in [2].

Finally, we study partially ordered sets labeled with k labels (k-posets) and their homomorphisms [4]. It turns out that the homomorphicity order of finite k-posets is a distributive lattice which is universal in the sense that every countable poset can be embedded into it. Our established connection between homomorphisms of k-posets and $\mathcal{C}$-minor relations when $\mathcal{C}$ is a clone of monotone functions with respect to a partial order on A in turn implies that such $\mathcal{C}$-minor partial orders also have this universal property.

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INCOMPLETE PROJECTION FAMILIES

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The study of the dimensions of the projections of measures invariant under the geodesic flow on the unit tangent bundle of an n-dimensional Riemann manifold leads to a 1-dimensional family of projections from $\mathbb{R}^{2(n-1)}$ to $\mathbb{R}^{n-1}$ ([2], [4]). Since the dimension of the collection of all projections from $\mathbb{R}^{2(n-1)}$ to $\mathbb{R}^{n-1}$ is $(n-1)^2$, this family is incomplete when $n \geq 3$. This phenomenon leads to the study of incomplete families of orthogonal projections from $\mathbb{R}^n$ to $\mathbb{R}^m$.

If some regularity is assumed on an incomplete collection of projections, bounds for the Hausdorff-dimension of the projected measures can be achieved in certain situations. In [1] we obtain sharp lower bounds in the cases of k-dimensional family of projections $\mathbb{R}^n \to \mathbb{R}^{n-1}$, 1-dimensional family of projections $\mathbb{R}^n \to \mathbb{R}^m$ and l-dimensional family of projections $\mathbb{R}^n \to \mathbb{R}$. Here $k, l, m \in \{1, \dots, n-1\}$.

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STOCHASTIC BINARY RELATIONS OF RANDOM VARIABLES AND PROCESSES

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Stochastic ordering is a fundamental technique for approximating random systems that are too complex to be analyzed exactly. In this talk I will sketch a definition of a stochastic binary relation that extends the notion of stochastic partial order into a relation between probability measures over two arbitrary state spaces. An application of Strassen's coupling theorem [1,2] then yields easily verifiable necessary and sufficient conditions for two Markov processes to preserve a given stochastic binary relation. These conditions turn out useful in approximating stationary distributions of vector-valued random walks, interacting particle systems, and queueing systems [3].

This talk is based on the work carried out in Centrum voor Wiskunde en Informatica (Amsterdam) and Eindhoven University of Technology.

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EFFECTIVE DYNAMICS OF LATTICE SYSTEMS IN A KINETIC SCALING LIMIT

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We consider weakly perturbed discrete wave-equations in a kinetic scaling limit: space and time are both scaled by a square of the coupling constant which is then taken to zero. Based on a formal perturbation expansion, it was first proposed by Peierls [1] that the dynamics of such systems can be studied using certain transport equations, here called phonon Boltzmann equations. We briefly discuss a recent conjecture [2], how phonon Boltzmann equations could appear in the kinetic scaling limit. To support the conjecture, we present two examples in which the scaling limit has proven to be mathematically tractable: for random mass perturbations in a three-dimensional lattice [3], and for equilibrium time-correlations of a discrete non-linear Schrödinger equation [4].

The talk is based on several joint works with Herbert Spohn.

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ON PRECONDITIONING OF DISSIPATIVE ACOUSTIC EQUATIONS

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Discretization of the linearized, time-harmonic Navier-Stokes equations that describe the compressional wave motion in a fluid (see for example [1]) leads to large systems of simultaneous linear equations for the unknown coefficients of velocity, pressure and temperature approximation. These linear systems can be difficult to solve efficiently using standard Krylov iteration methods. In this talk we describe recent advances in the development of block preconditioning techniques, with preconditioners derived from the Schur complement reduction [2].

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NONPARAMETRIC METHODS FOR MULTIVARIATE LOCATION PROBLEMS WITH INDEPENDENT AND CLUSTER CORRELATED OBSERVATIONS

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The aim of this research was to develop efficient nonparametric multivariate methods for independent and identically distributed (i.i.d.) observations and for cluster correlated observations. The first part deals with spatial sign and spatial rank methods, and their affine invariant and equivariant extensions, for the one-sample and the several samples multivariate location problem with i.i.d. observations. The second part focuses on the one-sample multivariate location problem with clustered data. Spatial sign methods, with their weighted generalizations and affine invariant and equivariant versions, are considered in this framework. The statistical properties (consistency, limiting distributions, limiting and finite sample efficiencies, robustness, computation) of the procedures are carefully investigated. It is shown that the spatial sign and rank methods have a competitive efficiency relative to the classical techniques, particularly if the data is heavy-tailed or clustered. The efficiencies and other statistical properties of the methods can be improved even further by weighting them in an optimal way. Furthermore, the methods are valid even without moment assumptions, and efficient when the underlying distribution deviates from normality or in the presence of outliers. The proposed procedures are easy to implement on statistical programming languages such as R or SAS/IML.

COMPOSITION OPERATORS AND ALEKSANDROV MEASURES

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Let ϕ be an analytic map taking the unit disc into itself. The linear operator $C_\phi : f \mapsto f \circ \phi$ is called the composition operator induced by ϕ. Such operators, acting on various spaces of analytic functions, have received much attention during the past few decades. The general idea has been to relate the function-theoretic properties of ϕ to the operator-theoretic properties of C_ϕ, such as compactness and spectra.

Every analytic map ϕ as above also determines a family of measures τ_α at the boundary of the unit disc, defined by the Poisson representation

$$\mathrm{Re}\frac{\alpha + \phi(z)}{\alpha - \phi(z)} = \int P_z \, d\tau_\alpha$$

for $|\alpha| = 1$. These measures are called the Aleksandrov measures of ϕ, and they have found many applications in modern function theory and related operator theory.

In my dissertation, principal topics are composition operators, Aleksandrov measures and their interaction. We use ideas stemming from the theory of composition operators to show that the Aleksandrov measures can be obtained as boundary values of (a refined version of) the Nevanlinna counting function of ϕ. Thus these measures have a subtle connection to the value distribution theory of analytic maps in the unit disc. In another direction, we use Aleksandrov measures as a tool to study the compactness of the difference of two composition operators on the classical Hardy spaces, extending earlier results for the case of a single composition operator.

On Linear Cryptanalysis

Kaisa Nyberg

The linear cryptanalysis is one of the most powerful cryptanalytic methods for conventional symmetric-key ciphers. It was presented in 1993 by Mitsuru Matsui, who used it to recover information of key bits in the DES block cipher. Since then it has been applied widely for constructing cryptanalytic attacks for both block ciphers and stream ciphers. Resistance against linear cryptanalysis is one of the most important design principles of modern ciphers. In this talk, the basic principles of linear cryptanalysis will be presented and some recent developments and cryptanalytic results will be described.

IDEAS AND EXPERIENCES IN LECTURE COURSES

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A lot of effort has been put to improving the studies of beginning students of mathematics in Helsinki. This has been done by intensive collaboration between students and teachers.

A good indicator of the results of the developments is the number of students passing the first "laudatur-level" courses (either the course 'measure and integration' or a certain basic course in teacher education.) This number was up to 2003 about 50 per year. This number includes also students for whom math is a minor subject. In 2005 the number was over 140 of whom over 100 were math majors. I seems that since this the last number will remain about 90 - 100 per year.

In this lecture several ideas about giving lectures and organizing a lecture course will be discussed.

EQUATIONS OVER SETS OF NATURAL NUMBERS

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Formal languages over a one-letter alphabet are just sets of nonnegative integers, and the current research on *language equations* [2] naturally leads to the important subcase of equations over sets of numbers. These are systems of equations of the form $X_i = \varphi_i(X_1, \ldots, X_n)$, where variables assume values of sets of nonnegative integers. Their right-hand sides may contain singleton constants, Boolean set-theoretic operations and the operation of pairwise sum of elements of two sets: $Y \boxplus Z = \{y + z | y \in Y, z \in Z\}$.

If the only Boolean operation is union, these systems represent context-free grammars over a unary alphabet; their least solutions are known to be ultimately periodic. If intersection is also allowed, they correspond to an extension of context-free grammars with conjunction: *conjunctive grammars* [3]. It is possible to represent a large class of non-periodic sets of numbers by solutions to such systems of equations: in particular, one can represent sets such as $\{2^n \mid n \geqslant 0\}$ and $\{2^{2^n} \mid n \geqslant 0\}$, and generally the growth of representable sets is not bounded by any recursive function [1]. Exponentially growing sets may still be represented by one-variable equations $X = \varphi(X)$, but in this case superexponential growth is no longer possible [4].

Another class of equations uses $\boxplus$-sum and complementation. Such systems can represent some non-periodic sets, but at the same time certain non-periodic sets of a simple structure were shown to be non-representable [5].

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HYPERBOLINEN FUNKTIOTEORIA GEOMETRISISSA ALGEBROISSA

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Kompleksilukujen ja kvaternioiden yleistys korkeampiin ulottuvuuksiin on nimeltään Cliffordin algebra. Kompleksisten funktioiden teoriaa voidaan yleistää Cliffordin algebrojen avulla korkeampiin ulottuvuuksiin. Näin saatua teoriaa kutsutaan *Cliffordin analyysiksi*. Keskeisenä työkaluna Cliffordin analyysissä on *Diracin operaattori*. Diracin operaattorin avulla määritellään Cliffordin analyysin tärkein funktioluokka, *monogeenisten funktioiden* luokka. Monogeenisia funktioita voidaan pitää funktioteorian analyyttisten funktioiden vastineina.

Monogeenisten funktioden teorialla on kuitenkin eräs heikkous, nimittäin potenssifunktiot eivät ole monogeenisia. Perusteltua onkin siis kysyä: Miten teoriaa tulisi muuntaa, että potenssit saataisiin mukaan? Vastauksen keksi 1990-luvun alussa professori Heinz Leutwiler Erlangenin yliopistosta. Leutwiler oivalsi, että siirtymällä euklidisesta metriikasta hyperboliseen metriikkaan, Diracin operaattori muuntuu ns. *hyperboliseksi Diracin operaattoriksi*. Potenssifunktiot saadaan upotettua mukaan teoriaan juuri hyperbolisen Diracin operaattorin avulla. Hyperbolisen Diracin operaattorin ympärille rakentuvaa korkeampiulotteista funktioteoriaa kutsutaan *hyperboliseksi funktioteoriaksi*.

Esitelmässä tarkastellaan Cliffordin algebroita, Cliffordin analyysiä ja hyperbolista funktioteoriaa erilaisissa Cliffordin algebroissa.

Subgroup growth and zeta functions of groups

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This will be an introductory talk about subgroup growth and zeta functions of groups.

Let G be an infinite group, and let $s_n(G)$ be a function that counts the number of subgroups up to index n in G. If G is finitely generated, then the function $s_n(G)$ produces a sequence of natural numbers. One natural question is to ask how does this sequence grow as n grows. For instance, $\mathbb{Z}$ has one subgroup for each index, so the function is $s_n(G) = n$ and the growth is linear in n. In other cases the growth can be polynomial, intermediate, exponential or even superexponential in n. The free group has superexponential growth.

One way to study this sequence is to consider $a_n(G)$ to be the precise number of subgroups of each index n, and encode this sequence in a generating function, in this case a Dirichlet series

$$\zeta_G(s) = \sum_{n=1}^{\infty} a_n(G) n^{-s}$$

and study the properties of this function. If we let G be a nilpotent group (almost abelian group), this zeta function has an Euler product similarly to the number theoretical zeta functions. In this lecture we shall explore further the properties of this function and what it tells about the lattice of subgroups inside nilpotent groups.

Esitelmä voidaan pitää myös suomeksi, missä tapauksessa tavoitteena on luoda uutta suomenkielistä terminologiaa ryhmäteorian alalle.

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FROM ENDOGENEOUS PRICES AND MICROSTRUCTURE TO BLACK–SCHOLES

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The conventional small-trader models of mathematical finance treat price processes of financial assets as given, *exogeneous* objects, with sophisticated sample path properties. On the other hand, on a microstructural level, asset prices change solely in response to finitely many asynchronous trades made by individual investors in the market. Clearly, in the interest of understanding the relation between "microscopic" behavior and aggregate-level phenomena in financial markets, it would be desirable to try to bridge this gap between these two rather differing viewpoints. In this talk, I will discuss how *limit theory for stochastic processes* could provide a link between microstructure-driven *endogeneous* prices and certain diffusion process models (e.g. that of Black and Scholes) of price dynamics. The talk is based largely on [1].

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WebALT virtuaalinen analyysin peruskurssi

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Professori Mika Seppälän luennoima virtuaalinen analyysin peruskurssi on ensimmäinen Helsingin yliopistossa järjestettävä matematiikan kurssi, joka toteutetaan tenttejä lukuun ottamatta kokonaan tietoverkossa WebALT teknologiaa käyttäen. Tämän kurssin toteutus voitti Helsingin yliopiston opetusteknologiapalkinnon vuonna 2007. WebALT (Web Advanced Learning Technologies) on kansainvälinen HY:n koordinoima EU eContent-projekti, joka toimi vuosina 2005 -2006. Projektin tuloksia hyödyntää WebALT yritys, jonka tehtävänä on edelleen kehittää projektissa kehitettyjä opetussisältöjä sekä teknisiä ratkaisuja.

Analyysin peruskurssin opetus on synkronista ja tapahtuu virtuaalisessa luokka-huoneessa. WebALT ratkaisu tarjoaa opiskelijoille tavanomaista kontaktiopetusta pa-remman ympäristön. Automaattisesti arvosteltavat tehtävät muodostavat järjestelmän tärkeimmän osan, automaattisen yksityisopetuksen.

Oppimateriaali on jaettu moduuleihin, jotka sisältävät seuraavat komponentit:

1. Kymmenen minuutin luento, jossa esitetään yksi matemaattinen käsite tai tulos.
2. Joukko ratkaistuja tehtäviä, missä esitetään esimerkkejä ko. tuloksen käytöstä.
3. Kotitehtäviä, joita käsitellään virtuaalisissa laskuharjoituksissa.
4. Laboratorioita, jotka havainnollistavat kyseistä käsitettä.
5. Sarja automaattisesti arvosteltavia WebALT MapleTA tehtäviä, joilla opiskelija voi osoittaa oppineensa ko. asian.

WebALT kursseja suunniteltaessa on tehty yhteistyötä prof. John Kellerin kanssa. Keller on verkko-opetuksen pedagogiikan johtava asiantuntija. Hän on kehittänyt ARCS mallin, jonka mukaisesti kurssimateriaalin ja verkko-opetuksen on:

1. Vangittava opiskelijan mielenkiinto hyvillä esimerkeillä: ATTENTION.
2. Esitettävä asia siten, että se on opiskelijalle RELEVANTTIA.
3. Tuettava opiskelijaa siten, että opiskelija saa varmuuden siitä, että hän on oppinut asiansa: CONFIDENCE.
4. Annettava opiskelijalle kätevä tapa osoittaa, että hän on oppinut asian: SATISFACTION.

Lisätietoja: http://webalt.math.helsinki.fi:8080/Calculus/, www.webalt.net, www.webalt.com

ROBUST STABILITY OF OBSERVERS

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In this presentation we consider robust stabilization of a distributed parameter system with an observer [2]. Our aim is to derive conditions under which the compensator stabilizes the system when the system operator used in the compensator differs from the original one.

We consider the system $\Sigma(A, B, C)$ where the pairs (A, B) and (A, C) are exponentially stablizable and detectable, respectively. It is well-known that if we choose operators F and K such that operators $A + BF$ and $A + KC$ generate exponentially stable C_0-semigroups, then the closed-loop system operator A_c generates an exponentially stable C_0-semigroup on $X \times X$. The replacement of the system operator A with an operator $\widetilde{A}$ in the observer can be seen as a perturbation of the closed-loop system operator A_c. Because of this, we can use theory on the preservation of exponential stablity of C_0-semigroups to derive conditions under which the new system operator $\widetilde{A}_c$ generates an exponentially stable C_0-semigroup.

We will present two separate sets of conditions for the stability of the new closed-loop system. The first conditions are given in terms of resolvent operators $R(\lambda, A + BF)$ and $R(\lambda, A + KC)$ and the second ones in terms of C_0-semigroups generated by $A + BF$ and $A + KC$. The theory used can be found in [3, 1, 4].

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The mathematics of gene assembly in ciliates

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Ciliates are an old and diverse group of unicellular eukaryotes. One of their unique features is that they have two types of functionally different nuclei, each present in multiple copies in each cell: micronuclei and macronuclei. The difference between the micronuclear and the macronuclear genome is striking, especially in Stichotrichs, on which we concentrate in this talk. While macronuclear genes are contiguous sequences placed in general on their own molecules, micronuclear genes are placed on long chromosomes, interrupted by stretches of non-coding material. Even more striking is that the micronuclear genes are split into several blocks (up to 44 of them in certain species), with the blocks arranged in a shuffled order, separated by non-coding material. Some blocks may even be inverted! At some stage during sexual reproduction, ciliates assemble the blocks in the orthodox order to yield the transcription able macronuclear gene. In this process, ciliates make use of some short specific sequences at the extremities of each MDS in the same way as pointers are used in computer science. Indeed, each coding block ends with a short sequence of nucleotides that is repeated in the beginning of the coding block that should follow it in the orthodox order. We refer to [3], [2] for more details on ciliates and on gene assembly.

We will give in the talk an overview of some of the research done on the mathematics of gene assembly in ciliates. Results include model building (in terms of permutations, strings and graphs), invariants, completeness, parallelism, and even suggestions on how gene assembly could be used for computing. We refer to [1] for a detailed presentation of some of these results.

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Robustisuus säätöteoriassa

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Robustisuudella tarkoitetaan systeemiteoriassa järjestelmän kykyä suoriutua tehtävistään ulkoisista häiriöistä ja järjestelmän parametrien muuntelusta huolimatta. Robusteilla rakenteilla on sellainen ominaisuus, että ne sitkeästi pyrkivät annettuun päämäärään huolimatta siitä, että niitä häiritään. Vaikka robustisuus on toivottava ominaisuus, useat ympärillämme olevat luonnonjärjestelmät eivät ole sellaisia: muutos esimerkiksi taivaankappaleiden massoissa johtaa muutoksiin planeettojen kiertoradoissa. Kiertoradat eivät ole robusteja, vaan koko planeettajärjestelmän käyttäytyminen muuttuu ajan kuluessa. Sen sijaan jotkin biologiset järjestelmät käyttäytyvät robustisti [1], [2].

Perinteisessä säätöteoriassa havaittiin jo varhain, että mittausignaali voitiin robustisti ohjata halutulle vakiotasolle, mikäli käytettiin takaisinkytkettyä säätäjää, joka sisälsi mittausignaalin ja asetusarvon erotuksen ajan suhteen integroivan termin ns. I-säätäjän.

1970-luvulla tehtiin tavallisten lineaaristen vakiokertoimisten järjestelmien teoriassa merkittävä havainto, josta käytetään nimitystä sisäisen mallin periaate (Internal Model Principle (IMP)) [3] . Sen mukaan järjestelmästä tehtävä mittaus saadaan ajan mukana robustisti seuraamaan annettua signaalia, mikäli takaisinkytketty säätäjä sisältää sopivan kopion signaalin generoivasta differentiaaliyhtälöstä. IMP on viime vuosina osittain yleistetty osittais- ja viiveyhtälöillä kuvattaville järjestelmille samoin kuin ääretönulotteisilla signaalingeneraattoreille [4].

Tässä esityksessä tarkastelemme niitä rakenteellisia tekijöitä, jotka takaavat lineaaristen ääretönulotteisten järjestelmien robustisuuden. Johdamme formaalisti ja yksinkertaisesti suljetun järjestelmän Sylvesterin yhtälön ja siitä sopivin oletuksin robustin säätäjän rakenteen.

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BIOMECHANICS IN SHOT PUT

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Leg push

In the research of biomechanics a theoretical developed analytical model of leg push is commonly used. The functioning and reliability of the model is assessed by comparing the results obtained by the model (for instance “ground reaction force – time” diagram) to the measured values of real leg push. The difficulty of this theoretical model is the fact that the complicated real leg push must be simplified into a form which fit the model. Then, among the factors which act in the real leg push one must choose the factors which essentially influence the leg push.

The idea of the research in Publication [2] is that it is first developed the model of jump with one leg and from this model it is developed the model of leg-pushing phase in shot put. Jump with one leg is the leg-pushing phase in shot put without pushing force. Therefore, the real leg-pushing phase in shot put is obtained by adding the pushing force to the model of jump with one leg.

Arm Push

In the Publication [3] A. Rahikainen has developed a model of arm push, in which it is assumed, that in muscle mechanics there is the maximum power P which is the largest power the muscle is able to produce within a certain velocity range. Then the muscle mechanics is not determined by the combined effect of forces in the system, but by the constant maximum power within a certain velocity range.

According to the theory motion proceeds from start to maximum velocity as follows:

1. Start phase.
2. The motion proceeds with constant maximum force F. The resulted moment M at the joint changes as the motion proceeds, because the length of moment arm r changes.
3. The motion proceeds with constant maximum power P to the highest speed.

Maximum power hypothesis in Phase 3 can be tested by fitting the equation of motion consistent with that Phase into angular velocity-time measurements. On the contrary maximum force hypothesis of Phase 2 cannot be tested by measurement fitting, because the length of the moment arm r is indeterminate, and therefore the moment M is also indeterminate, and the moment determines the angular velocity.

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TIME-SERIES ANALYSIS OF MAGNETOMETER DATA WITH STATISTICAL INVERSE METHODS

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In this talk we consider time-series analysis with statistical inverse methods with applications to magnetometric measurements. First of all, we show the regular deconvolution problem of a magnetometric measurements. Thereafter we consider combining signals from different kind of magnetometers. We also consider representation of a priori information in a unified form as correlation length controllable priors. We apply the developed priors to an an example of translation of a grid, which is very much needed tool in magnetometer data analysis

Secondly we consider how to measure the unknown directions of magnetometers. These directions are generally known according to the manufacturer's leaflet. However, in practice these directions seem to be something else than stated in the manufacturer's leaflet. Therefore, in order to make for example some kind of combination of signals from different magnetometers, these directions need to be known accurately.

In the third part of talk, we show three novel methods for determining the impulse response of pulsation magnetometers. The first method is based on transmitting a sweep signal from an external coil. In the second method we use white noise instead of a sweep signal. In the third method, we consider how well a ground to cloud lightning mimics Dirac delta impulse and how can we deduce the impulse response of a magnetometer based on the lightning.

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OPERATOR THEORY IN DIFFRACTIVE OPTICS

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In diffractive optics the two principal optical elements are lenses and free propagation. Expressed in terms of linear operators on $L^2(\mathbb{R}^n)$, the integral operator

$$U_d f(x) = e^{-\pi i n/4} d^{-n/2} (2\pi)^{-n/2} \int_{\mathbb{R}^n} e^{i(x-y)\cdot(x-y)/2d} f(y)\, dy, \quad f \in L^2(\mathbb{R}^n)$$

corresponds to free propagation in a medium by a distance $d > 0$. The operator

$$V_P f(x) = e^{-iPx\cdot x/2} f(x),$$

with P a symmetric matrix, corresponds to a lens [1]. Diffractive optical systems are thus described by the group generated by these two types of operators.

We will show with the help of the metaplectic representation and properties of the symplectic group that the lenght of any group element is bounded. In fact, the number of certain elementary constituents is at most 12 or 36 depending on a criterion on symplectic matrices.

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ÄÄRELLISET YKSINKERTAISET RYHMÄT JA KAHDEN INVOLUUTION TULOT

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Ryhmää kutsutaan yksinkertaiseksi, jos sillä on täsmälleen kaksi normaalia aliryhmää: ryhmä itse sekä neutraalialkion muodostama aliryhmä. Useiden vuosikymmenien ajan matemaatikot ovat pyrkineet luokittelemaan kaikki äärelliset yksinkertaiset ryhmät ja tällä hetkellä ollaan vakuuttuneita siitä, että luokittelussa on onnistuttu. Väitteen todistus on tuhansien sivujen pituinen, ja se on yksi ryhmäteorian merkittävimmistä saavutuksista.

Involuutiot puolestaan ovat alkioita, joiden kertaluku on kaksi. Yritän parhaillaan selvittää, missä äärellisissä yksinkertaisissa ryhmissä kaikki alkiot ovat kahden involuution tuloja. Monien ryhmien tapauksessa vastaus tiedetään, mutta muutamien osalta ongelma on vielä ratkaisematta.

Esitelmässäni kuvailen, miltä äärelliset yksinkertaiset ryhmät näyttävät, sekä kerron lyhyesti tutkimuksestani.

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Fraktionaalisen Brownin liikkeen lokaali riippumattomuus

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Olkoon $(X_t)_{-\infty<t<\infty}$ fraktionaalinen Brownin liike Hurstin indeksillä $H \in (0,1)$. Merkitään $\sigma_{(t,t')}$:llä sigma-algebraa, jonka generoivat erotukset $X_s - X_{s'}$ kun $s, s' \in (t,t')$. Olkoot t_1 ja t_2 erisuuria ajanhetkiä. Näytämme, että $\sigma_{(t_1-\varepsilon,t_1+\varepsilon)}$ ja $\sigma_{(t_2-\varepsilon,t_2+\varepsilon)}$ ovat asymptoottisesti riippumattomia kun $\varepsilon \searrow 0$. Toisin sanoen, *lokaali riippumattomuus* on voimassa fraktionaaliselle Brownin liikkeelle. Osoitamme tämän huomattavasti vahvemmassa mielessä: Shannonin keskinäinen informaatio kyseisten sigma-algebrojen välillä lähestyy nollaa kun $\varepsilon \searrow 0$. Tarkastelemme myös kvantitaativisia tuloksia.

Esitelmä perustuu yhteistyöhön Ilkka Norroksen (VTT) kanssa.

Optimal stopping – applications, theory and history

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Abstract

In this talk we consider first three classical applications of optimal stopping:

- Bayesian sequential (statistical) hypothesis testing problem,
- Secretary problem,
- Pricing American options.

After this, some main points of the theory of optimal stopping are presented. We focus on the Markovian case and discuss:

- the concept of the smallest excessive majorant of the reward function,
- a verification theorem for the solutions obtained using the principle of smooth fit (diffusion case),
- a new verification theorem based on the Riesz representation of excessive functions which is valid for fairly general Hunt processes; in particular, for Lévy processes (joint work with Ernesto Mordecki).

The talk is concluded with a short discussion on the historical development of the theory of optimal stopping.

CARLEMAN ESTIMATES AND ANISOTROPIC INVERSE PROBLEMS

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We consider the imaging of anisotropic materials by electrical measurements. This inverse problem arises in Electrical Impedance Tomography (EIT), which has been proposed as a diagnostic method in medical imaging and nondestructive testing. The mathematical model is the anisotropic Calderón problem, which consists in determining a matrix of coefficients in an elliptic equation from boundary measurements of solutions.

The anisotropic Calderón problem can be formulated in geometric terms, as the recovery of a Riemannian metric from Cauchy data of harmonic functions on a manifold. Our approach is based on Carleman estimates. We characterize those Riemannian manifolds which admit a special limiting Carleman weight. By using these weights, we construct special harmonic functions and prove uniqueness results in anisotropic inverse problems for a class of Riemannian manifolds.

This is joint work with D. Dos Santos Ferreira (Paris 13), C. Kenig (Chicago), J. Sjöstrand (École Polytechnique), and G. Uhlmann (Washington).

Between Freedom and Coercion: The emergence of costly punishment

Karl Sigmund

Abstract: It is well-known that team efforts and public goods present a social dilemma. Individuals aiming to maximise their income will not cooperate. This changes for the better if incentives are used (rewards and punishment). But a sanctioning system is itself a public good, and the emergence of such a system is a widely discussed game theoretic problem. In this talk a solution is offered, based on evolutionary game theory in finite populations. Interestingly, it only works if participation is voluntary, rather than compulsory.

MATEMAATTISET MENETELMÄT HENKILÖKOHTAISESSA PAIKANNUKSESSA

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Nykyiset kaupasta saatavat gps-paikantimet toimivat hyvin maanteillä, merellä ja ilmassa, mutta kaupungissa ja varsinkin sisätiloissa tulee ongelmia, kun näköyhteys paikannussatelliitteihin katkeaa. Esimerkiksi matkapuhelimissa ja kämmentietokoneissa on kuitenkin jo nyt saatavissa eri radioverkkojen ja mahdollisten muiden antureiden kautta paljon muutakin informaatiota, jota voidaan käyttää paikannukseen. Eräs haaste näiden tietojen käytössä on se, ettei niitä ole alun perin tarkoitettu paikantamiseen.

Kun sekä mittaukset, vastaanottimen dynamiikka, että mahdolliset muut rajoitteet mallinnetaan ehdollisina todennäköisyystiheyksinä, voidaan paikannusongelma muotoilla tehtäväksi jossa koitetaan ratkaista paikan (ja muiden kiinnostavien suureiden) todennäköisyysjakauma ehdollistettuna saaduilla mittauksilla. Tehtävällä on tarkka ratkaisu, rekursiivinen Bayesilainen suodatin, mutta sille ei muutamia erikoistapauksia lukuunottamatta ole suljetun muodon ratkaisua. Lineaaris-Gaussisen erikoistapauksen tarkasti ratkaisevasta Kalmanin suodattimesta on kehitelty monenlaisia laajennuksia, ja viime vuosina uudenlaiset laskennallisesti huomattavasti raskaammat hilaintegrointiin tai Monte Carlo -simulointiin perustuvat menetelmät ovat alkaneet tulla käyttökelpoisiksi.

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LOCAL CONTINUITY (FOR STOPPING TIMES)

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We propose the concept of Local Continuity that is closely related to directional continuity.

Definition. Let X and Y be, say, metric spaces. A function $f : X \to Y$ is *locally continuous* at point $x \in X$ if one can find an open set $U_x \subset X$ such that

(i) $x \in \bar{U}_x$,

(ii) if $x_n \to x$ in U_x then $f(x_n) \to f(x)$ in Y.

The set U_x, the *local continuity set* of f at x, tells the direction of continuity. If U_x can be chosen to contain x then f is continuous at x.

We do not know if local continuity is a new or even interesting concept. The concept was conceived during our study [1] where we considered non-semimartingale pricing models that have non-trivial quadratic variation and a certain "small-ball property". It turned out that in these models one cannot do arbitrage with strategies that are continuous in terms of the spot and some other economic factors such as the running minimum and maximum of the stock. Unfortunately, this result does not extend to even simple strategies, when stopping times are involved. The reason is obvious: Stopping times are typically not continuous in the stock price. However, local continuity turns out to be just what we need to prove our theorems, and the author is not aware of any reasonable stopping times that are not locally continuous.

The talk is based on an ongoing joint work with C. Bender (Technical University of Braunschweig), D. Gasbarra (University of Helsinki), and E. Valkeila (Helsinki University of Technology).

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BILLIARDS AND STATISTICAL PROPERTIES OF TORAL AUTOMORPHISMS

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Imagine a ball rolling on an infinite table, frequently colliding elastically with round obstacles at fixed positions. This is the so-called Lorentz gas, or Sinai billiards. It is well known that if the array of obstacles is periodic and of "finite horizon" the system is ergodic and enjoys strong statistical properties such as the central limit theorem [1] and exponential decay of correlations [2,3].

The Lorentz gas with randomly positioned obstacles stands as a challenge. We introduce a toy model for the latter, called the random toral automorphism, and discuss its statistical properties.

The talk is based on joint work with Arvind Ayyer and Carlangelo Liverani; see [4,5].

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WHICH MEASURES ARE PROJECTIONS OF PURELY UNRECTIFIABLE HAUSDORFF MEASURES

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In this talk we will give a necessary and sufficient condition for a measure on the real line to be an orthogonal projection of one-dimensional Hausdorff measure restricted to some purely 1-unrectifiable planar set.

The results have been obtained in collaboration with M. Csörnyei.

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On a short sum involving an inner product of a holomorphic cusp form and a Maass form

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We confine ourselves to cusp forms for the full modular group Γ. A *holomorphic cusp form* $F(z) : \mathbb{H} \to \mathbb{C}$ of weight k can be represented by its Fourier series

$$F(z) = \sum_{n=1}^{\infty} a(n) e(nz).$$

Maass wave forms

$$u_j(z) = y^{1/2} \sum_{n \neq 0} \rho_j(n) K_{i\kappa_j}(2\pi |n| y) e(nx)$$

constitute an orthonormal set of non-holomorphic cusp forms arranged so that the corresponding parameters κ_j determined by the eigenvalues $1/4 + \kappa_j^2$ lie in an increasing order. We write

$$c_j = (u_j(z), y^k |F(z)|^2)$$

for the Petersson inner product including the two cusp forms described above. Jutila has proved an estimation

$$\sum_{\kappa_j \leq K} |c_j|^2 \exp(\pi \kappa_j) \ll K^{2k+\varepsilon}$$

in his article [1]. We have now expanded this theorem by proving the expected estimation for the sum over the short interval $K \leq \kappa_j \leq K + K^{1/3}$. Our result gives better local knowledge of the behavior of the error term in the additive divisor problem, and likewise of the asymptotic formula for the fourth moment of Riemann's zeta-function.

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ON TOEPLITZ AND HANKEL OPERATORS IN THE BERGMAN SPACE A^1

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The theory of Toeplitz and Hankel operators is quite well understood for Hardy spaces on the unit disc $\mathbb{D}$ of $\mathbb{C}$. In the case of Bergman spaces $A^p(\mathbb{D})$, $1 \le p < \infty$, even the most basic questions like boundedness have only been partially answered.

We study Toeplitz and Hankel operators, denoted by T_a and H_a respectively, with bounded symbols a belonging to the class $BMO_{\partial \log}(\mathbb{D})$, acting on the space $A^1(\mathbb{D})$. We prove a standard boundedness result. As for compactness and related matters, we assume that a is continuous on the closed disc and has a vanishing hyperbolic logarithmic mean oscillation. Using a rather delicate approximation lemma concerning the boundary behaviour of symbols, we show that

- $H_a : A^1(\mathbb{D}) \to L^1(\mathbb{D})$ is compact,
- $T_a : A^1(\mathbb{D}) \to A^1(\mathbb{D})$ is Fredholm, if $a(t) \neq 0$ for every $t \in \partial\mathbb{D}$, and
- $T_a : A^1(\mathbb{D}) \to A^1(\mathbb{D})$ is compact, if $a(t) = 0$ for every $t \in \partial\mathbb{D}$.

This is a joint project with Jani Virtanen (University of Helsinki).

MATEMATIIKAN OPETTAJANKOULUTUKSEN HAASTEITA JA MAHDOLLISUUKSIA

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Millaista matematiikkaa aineenopettajakoulutuksen matematiikan kursseilla tulisi opettaa? Millainen rooli matematiikan didaktiikan opinnoilla aineenopettajantutkinnossa tulisi olla? Tässä esityksessä tarkastellaan matematiikan opettajankoulutuksen kehittämistä toisaalta aineenopettajaksi opiskelevien nykyisen osaamisen näkökulmasta ja toisaalta opiskelijoiden omien matematiikan opettamiseen liittyvien käsitysten pohjalta.

Esityksessä tarkastellaan mm. yhtälön käsitteen oppimistuloksia ja kolmea oppimisen metaforaa, jotka kuvaavat yleistä oppimiskäsityksen kehittymistä kasvatustieteen piirissä. Lisäksi esitellään tuloksia kyselytutkimuksesta, jossa matematiikan aineenopettajaksi opiskelevat kertovat käsityksistään siitä, millaista matematiikkaa tulevaisuuden koulussa tulisi opettaa. Johtopäätöksenä näistä tutkimuksista voidaan todeta, että matematiikan aineenopettajakoulutusta tulisi merkittävästi muuttaa sekä rakenteellisesti että sisällöllisesti, jotta koulutus vastaisi opiskelijoiden ja jo työssä olevien aineenopettajien odotuksia ja tarpeita.

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AXIOMATIC EXTENSIONS OF MONOIDAL LOGIC

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When Ulrich Höhle introduced Monoidal logic [2], his purpose was to outline a common framework for a diversity of monoidal structures which constitute the basis of various non–classical logics. Indeed, Monoidal logic connects Intuitionistic logic, Lukasiewicz logic and (commutative) Linear logic, i.e. these logics are axiomatic extension of Monoidal logic. The main tool to prove the completeness of Monoidal logic and its various axiomatic extensions is to construct a canonical model and show that if a formula is not provable, then it is not valid in this model. The canonical model is obtained by constructing first the corresponding Lindenbaum algebra and then its MacNeille completion.

In this paper we define some new axiomatic extensions of Monoidal logic. Our primary motivation is the following. Based on Kolmogorov's idea [3], *Glivenko Theorem* asserting that if a propositional formula admits a classical proof, then its *double negation* admits an intuitionistic proof, and hence we have a classical proof of a propositional formula if and only if we have an intuitionistic proof of its double negation. We extend this result by presenting the widest such axiomatic extension of Monoidal logic whose negative part constitutes Lukasiewicz logic. This logic will be called *Semi–divisible logic*. Moreover, we show how *Monoidal t-norm based logic* [1] is related to Monoidal logic. Our approach is deeply algebraic.

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PSEUDO-DIFFERENTIAL OPERATORS AND SYMMETRIES

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We study pseudo-differential operators globally on compact Lie groups, without resorting to local charts. A pseudo-differential operator can be presented as a convolution operator valued mapping on the group, and we obtain a full global symbol and global calculus. As an example, we investigate analysis on the 3-dimensional sphere.

This is a joint work with M. Ruzhansky (Imperial College London).

Composition operators on spaces of vector-valued functions

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Let D be the unit disk in the complex plane and φ an analytic self-map of D. The theory of the composition operators C_φ, where $f \mapsto f \circ \varphi$, on various Banach spaces of scalar-valued analytic functions on D (such as e.g. the Hardy and Bergman spaces, $BMOA$) is very extensive. Here I will describe some recent work about properties of $f \mapsto f \circ \varphi$ on certain Banach spaces consisting of *vector-valued* analytic functions $f : D \to X$, where X is an infinite dimensional complex Banach space (see e.g. [LST], [BDL], [L1], [L2], [LT]).
The topics discussed will include: (1) qualitative properties of C_φ, such as weak compactness, (2) a characterization of the boundedness of $C_\varphi : wH^p(X) \to H^p(X)$ in terms of the Hilbert-Schmidt norm of $C_\varphi : H^2 \to H^2$ for $2 \leq p < \infty$, [LTW]. (Above $wH^p(X)$ is the weak X-valued Hardy space defined by requiring that $\sup_{\|x^*\| \leq 1} \|x^* \circ f\|_{H^p} < \infty$, where $x^* \in X^*$.) This result was motivated by a question of Kaijser.

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REGION OF VARIABILITY FOR CERTAIN CLASSES OF UNIVALENT FUNCTIONS SATISFYING DIFFERENTIAL INEQUALITIES

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For complex numbers α, β and $M \in \mathbb{R}$ with $0 < M \leq |\alpha|$ and $|\beta| \leq 1$, let $\mathcal{B}(\alpha, \beta, M)$ be the class of analytic and univalent functions f in the unit disk $\mathbb{D}$ with $f(0) = 0$, $f'(0) = \alpha$ and $f''(0) = M\beta$ satisfying $|zf''(z)| \leq M$, $z \in \mathbb{D}$. Let $\mathcal{P}(\alpha, M)$ be the another class of analytic and univalent functions in $\mathbb{D}$ with $f(0) = 0$, $f'(0) = \alpha$ satisfying $\mathrm{Re}\,(zf''(z)) > -M$, $z \in \mathbb{D}$, where $\alpha \in \mathbb{C} \setminus \{0\}$, $0 < M \leq 1/\log 4$. For any fixed $z_0 \in \mathbb{D}$ and $\lambda \in \overline{\mathbb{D}}$ we shall determine the region of variability V_j $(j = 1, 2)$ for $f'(z_0)$ when f ranges over the class $\mathcal{S}_j$ $(j = 1, 2)$, where $\mathcal{S}_1 = \{f \in \mathcal{B}(\alpha, \beta, M) : \ f'''(0) = M(1 - |\beta|^2)\lambda\}$ and $\mathcal{S}_2 = \{f \in \mathcal{P}(\alpha, M) : \ f''(0) = 2M\lambda\}$.

These results have been done in collaboration with S. Ponnusamy and M. Vuorinen

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THE STONE-ČECH COMPACTIFICATION OF A TOPOLOGICAL GROUP

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Compact right topological semigroups have remarkable algebraic properties. We are interested particularly in those obtained as semigroup compactifications (see e.g. [2]) such as the Stone-Čech compactification βG of a discrete group G. The Stone-Čech compactification βG of an infinite discrete group (or more generally a semigroup) G can be turned into a semigroup by an operation, continuous *only* on one side, extended from G such that G is contained in the topological centre of βG. This operation can be described in many ways depending on how we regard βG.

We shall consider the possibility of defining the operation on βG for general topological groups, G belonging to the topological centre of βG. This problem was completely settled by Baker and Butcher for locally compact groups in [4], where they proved that βG is a semigroup as above if and only if G is either discrete or compact. We shall generalize the result of Baker and Butcher in [4], for a wide class of non-locally compact topological groups, excluding only the class so-called P-groups.

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BELTRAMI-KENTTIEN SIRONTAA

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Sirontateoriassa tutkitaan aallon siroamista. Kun homogeenisessa aineessa, tai tyhjiössä, etenevä aalto osuu epähomogeenisuuteen, tämä aiheuttaa aaltoon häiriötä, jota sanotaan sironneeksi aalloksi. Epähomogeenisuutta kutsutaan sirottajaksi. Se voi olla esimerkiksi kala vedessä, lentokone ilmassa, kasvain kudoksessa tai halkeama kattopalkissa. Aallot voivat olla esimerkiksi ääniaaltoja tai sähkömagneettisia aaltoja.

Suorassa sirontaongelmassa tehtävänä on ratkaista sironnut aalto, kun alkuperäinen aalto ja sirottaja tunnetaan. Käytännössä tämä tarkoittaa tilannetta mallittavan osittaisdifferentiaaliyhtälösysteemin ratkaisemista - teoreettisesti ja numeerisesti. Koska luonnossa aalto aina siroaa jotenkin, niin yleensä suora ongelma on matemaattisestikin ratkeava, kunhan malli vain kuvaa luontoa riittävän hyvin.

Sironnan inversio-ongelmassa, eli käänteisessä sirontaongelmassa, tehtävänä on määrittää sirottaja, kun tunnetaan kutakin alkuperäistä aaltoa vastaava sironnut aalto. Tämänkaltainen tilanne tulee tyypillisesti vastaan mittauksissa: lähetetään aaltoja kiinnostavalle alueelle, mitataan sirontaa, ja halutaan päätellä, mitä kiinnostavalla alueella on. Sironnan inversio-ongelmia on itseasiassa hyvin monenlaisia riippuen siitä, miten sirontaa mitataan ja mitä sirottajasta halutaan selvittää. Yleensä inversio-ongelmien minkäänlainen ratkeavuus ei ole lainkaan itsestäänselvää.

Singulaaristen lähteiden menetelmällä on mahdollista ratkaista sirottajan muoto tietynlaisen mittausdatan avulla. Menetelmän idea on konkreettinen: Kuljet huudellen ympäriinsä ja kuuntelet omaa ääntäsi. Kun tulet lähelle seinää, äänesi heijastus seinästä kasvaa. Näin saat seinän paikan selville. Singulaaristen lähteiden menetelmässä tällainen "liikuteltava lähde" muodostetaan laskennallisesti mittausdatan avulla. Myös "heijastuksen kuunteleminen" suoritetaan laskennallisesti mittausdatan avulla. Sirottajan lähellä heijastus kasvaa suureksi ja näin saadaan sirottajan reuna selville.

Väitöstyössä tarkastellaan lineaaristen Beltrami-kenttien suoraa ja käänteistä sirontaongelmaa. Vektorikenttä on lineaarinen Beltarmi-kenttä, jos sen pyörteisyys on verrannollinen kenttään itseensä paikasta riippumattomalla vakiolla. Tutkittavassa inversio-ongelmassa oletetaan sironnut kenttä tunnetuksi kaukana sirottajasta ja tehtävänä on määrittää sirottajan muoto. Työssä todistetaan sekä suoran että kyseisen käänteisen sirontaongelman ratkeavuus Beltrami-kentille ja lasketaan numeerisia esimerkkejä.

Three-dimensional Competitive and Competitor-competitor-mutualist Lotka-Volterra Systems

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For three-dimensional competitive Lotka-Volterra systems, Zeeman (1993) identified 33 stable nullcline equivalence classes. Among these, only classes 26-31 may have limit cycles. Hofbauer and So (1994) conjectured that the number of limit cycles is at most two for these systems. They also conjectured that, in the case of a heteroclinic cycle on the boundary of the carrying simplex, condition A (i.e. the heteroclinic cycle is neutrally stable) might be replaced by condition B (i.e. second focal value vanishes). In this talk we construct four limit cycles for a three-dimensional competitive Lotka-Volterra system with a heteroclinic cycle. Furthermore, we show that condition A cannot be replaced by condition B.

We also give some new results for the competitor-competitor-mutualist Lotka-Volterra systems.

The results have been done in collaboration with M. Gyllenberg, and with M. Gyllenberg and Y. Wang.

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TOMOGRAPHY WITH UNKNOWN PROJECTION DIRECTIONS

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A compactly supported integrable function in the plane is uniquely determined by the values of its line integrals over all lines through the support of the function. This is a classical fact from the theory of the Radon transform. We consider the following variant of this problem. Suppose one knows the line integral projections of an unknown function to a number of directions, i.e. one knows the values of all line integrals parallel to these directions, but that these directions itself are unknown. Does this data determine the function uniquely? Alternatively, does the data determine the projection directions uniquely?

This kind of problem of tomography with unknown projection directions appears in certain practical applications. For instance, in magnetic resonance imaging there may be uncertainty in the projection directions due to involuntary motion of the patient. The three-dimensional version of this problem occurs in cryo electron microscopy of viral particles, where the projection directions may be completely unknown due to the random orientations of the particles being imaged.

The problem of tomography with unknown projection directions is related to some algebraic geometric properties of a certain system of homogeneous polynomials. We review this connection, and consider a recent uniqueness result for the problem [1]. The main result is that for sufficiently asymmetric functions the projections from infinitely many unknown directions determine the function uniquely.

The results have been done in collaboration with Lars Lamberg.

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ON POWERS AND RESOLVENTS OF LINEAR OPERATORS

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Abstract. We intend to discuss various Kreiss-type conditions related to the behaviour of powers of linear operators.