In[1]:=
```
SetDirectory["~/kappaLib/"];
<< kappaLib-1.2.m
<< helper.m
```

```
Loading KappaLib v1.2

Loading helper.m..
```

- **Verify the identities used to prove Theorem 4.5**
- **We assume that A,B are arbitrary antisymmetric (2,0)-tensors. Say**

  A = 1/2 A^ij d/dx^i ∧ d/dx^j

  and coefficients A^ij are given by matrix A. Similarly for B.

In[4]:=
```
A = emMatrix["A", 4, Structure → "AntiSymmetric"];
B = emMatrix["B", 4, Structure → "AntiSymmetric"];
A // MatrixForm
B // MatrixForm
```

Out[6]//MatrixForm=

$$\begin{pmatrix} 0 & A12 & A13 & A14 \\ -A12 & 0 & A23 & A24 \\ -A13 & -A23 & 0 & A34 \\ -A14 & -A24 & -A34 & 0 \end{pmatrix}$$

Out[7]//MatrixForm=

$$\begin{pmatrix} 0 & B12 & B13 & B14 \\ -B12 & 0 & B23 & B24 \\ -B13 & -B23 & 0 & B34 \\ -B14 & -B24 & -B34 & 0 \end{pmatrix}$$

In[8]:=
```
(* kappa = general (2,2)-tensor *)
kappa = emGeneralKappa["k"];
```

In[9]:=
```
(* contract kappa with a bivector from the left *)
contract[biv_, kappa_] := Table[
  1 / 2 Sum[
    biv[[i]][[j]] emReadNormal[kappa, a, b, i, j]
    ,
    {i, 1, 4}, {j, 1, 4}
   ]
  ,
  {a, 1, 4}, {b, 1, 4}
 ]
```

- **Identities**

In[10]:=
```
delta = emPoincare[emBiProduct[density, A, B]] - emBiProduct[density, B, A];
Union[Flatten[Simplify[delta]]]
```

Out[11]= {0}

In[12]:=
```
LHS = emCompose[emBiProduct[density, A, B], kappa];
RHS = emBiProduct[density, A, contract[B, kappa]];
Union[Flatten[Simplify[RHS - LHS]]]
```

Out[13]= {0}

In[14]:=
```
LHS = emCompose[emBiProduct[density, A, B], kappa];
RHS = emBiProduct[density, A, contract[B, kappa]];
Union[Flatten[Simplify[RHS - LHS]]]
```

Out[15]= {0}

In[16]:=
```
LHS = emCompose[kappa, emBiProduct[density, A, B]];
RHS = emBiProduct[density, contract[A, emPoincare[kappa]], B];
Union[Flatten[Simplify[RHS - LHS]]]
```

Out[17]= {0}

In[18]:=
```
LHS = emCompose[emBiProduct[density, A, B], emBiProduct[density, B, A]];
RHS = emTrace[emBiProduct[density, B, B]] emBiProduct[density, A, A];
Union[Flatten[Simplify[RHS - LHS]]]
```

Out[19]= {0}

In[20]:=
```
LHS = emPoincare[emPoincare[kappa]];
RHS = kappa;
Union[Flatten[Simplify[RHS - LHS]]]
```

Out[22]= {0}

- **Local expression for density bar(B) \otimes B.**

In[23]:=
```
FullSimplify[emTrace[emBiProduct[density, B, B]]]
```

Out[23]= 4 (B14 B23 - B13 B24 + B12 B34) density