In[1]:=
```
SetDirectory["~/KappaLib/"];
<< kappaLib-1.2.m
<< helper.m
```

```
Loading KappaLib v1.2

Loading helper.m..
```

- **Define Metaclass I**

In[4]:=
$$\texttt{kappa = emMatrixToKappa}\left[\begin{pmatrix} a1 & 0 & 0 & -b1 & 0 & 0 \\ 0 & a2 & 0 & 0 & -b2 & 0 \\ 0 & 0 & a3 & 0 & 0 & -b3 \\ b1 & 0 & 0 & a1 & 0 & 0 \\ 0 & b2 & 0 & 0 & a2 & 0 \\ 0 & 0 & b3 & 0 & 0 & a3 \end{pmatrix}\right];$$

```
kappa = kappa //. {b3 → b2, a3 → a2};
```

Let us first observe that the Hodge star operator can be rewritten using the emQMedium routine. This simplifies the expressions as we can specify the inverse of g and keep Sqrt(Abs(Det(g))) as a symbolic variable

In[6]:=
```
Metric = emMatrix["g", 4, Structure → "Symmetric"];
hodge = emHodge[Metric];
hodgeAlt = emQMedium[Sqrt[Abs[Det[Metric]]], Inverse[Metric]];
```

In[9]:=
```
Union[Flatten[hodge - hodgeAlt]]
```

Out[9]= {0}

Case: a1 = a2

- **Define bivector A and metric g**

In[10]:=
$$\texttt{Abivector = }\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix};$$

```
Metric = Inverse[DiagonalMatrix[{1, -1, -Psi / b2, -Psi / b2}]];
```

- **Check that metric has Lorentz signature**

In[12]:=
```
Det[Metric]
```

Out[12]= $-\frac{b2^2}{Psi^2}$

- **Formulate equations that should be satisfied**

In[13]:=
```
kappaAlt = C1 emQMedium[SqrtAbsDetG, Inverse[Metric]] +
    emBiProduct[rho, Abivector, Abivector] + C2 emIdentityKappa[];
eqs = Union[Flatten[FullSimplify[kappa - kappaAlt]]];
```

- **Show that the equations are satisfied for a suitable choice of parameters:**

In[15]:=
```
sub = {
    a2 → a1,
    C1 -> -1 / SqrtAbsDetG b2^2/Psi ,
    C2 → a2,
    rho → 1 / 2 (b2 ^ 2 - b1 ^ 2) / b1,
    Psi → b2 ^ 2 / b1
   };
show[simp[eqs //. sub]]
```

Out[16]//MatrixForm=

$(1 \;:\; 0)$

Case: a1 != a2

- **Define A and metric g**

In[17]:=

$$\text{Abivector} = \begin{pmatrix} 0 & \text{SqrtAbsXi} & 0 & 0 \\ -\text{SqrtAbsXi} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\text{a1}-\text{a2}}{2\ \text{rho SqrtAbsXi}} \\ 0 & 0 & -\frac{\text{a1}-\text{a2}}{2\ \text{rho SqrtAbsXi}} & 0 \end{pmatrix};$$

```
Metric = Inverse[DiagonalMatrix[{1, -1, -Psi / b2, -Psi / b2}]];
```

- **Formulate equations that should be satisfied**

In[19]:=
```
kappaAlt = C1 emQMedium[SqrtAbsDetG, Inverse[Metric]] +
    emBiProduct[rho, Abivector, Abivector] + C2 emIdentityKappa[];
eqs = Union[Flatten[FullSimplify[kappa - kappaAlt]]];
```

- **Show that the equations are satisfied for a suitable choice of parameters:**

In[21]:=
```
sub = {
    C1 → - 1/SqrtAbsDetG b2 ^ 2 / Psi,
    C2 → a2,
    rho → SgnXi
   };
show[simp[eqs //. sub]]
```

Out[22]//MatrixForm=

$$\begin{pmatrix} 1 & : & 0 \\ 2 & : & \text{b1} - \frac{\text{b2}^2}{\text{Psi}} - 2\ \text{SgnXi SqrtAbsXi}^2 \\ 3 & : & -\text{b1} + \text{Psi} - \frac{(\text{a1}-\text{a2})^2}{2\ \text{SgnXi SqrtAbsXi}^2} \end{pmatrix}$$

- **Since Sgn(b1)=Sgn(Psi) the last two equations are equivalent with**

  Xi = 1/2(beta1 - beta2^2/beta1)

  -beta1 + Psi - (alpha1 - alpha2)^2/(2Xi) = 0

- **The first of these is the definition of Xi**
- **The latter holds since Psi satisfies**

  1/beta2 Psi^2 - D3 Psi + beta2 = 0

where D3 is defined as above.