```
SetDirectory["~/KappaLib/"];
<< kappaLib-1.2.m
<< helper.m
```

```
Loading KappaLib v1.2

Loading helper.m..
```

Define 4-parameter medium kappa such that

1) each kappa is algebraically decomposable

2) beta^2-alpha gamma = 1

3) Fresnel surface always factorises into two linear factors and a quadratic form. For suitable parameters, the quadratic form can have a Lorentz signature

4) The nonlinear equation for D has no solution.

```
kappa = emMatrixToKappa[( tau + C1   -1    0    0    0      0
                             C1     tau    1    0    0      0
                              1      0    tau   0    0      0
                              0      0     1   tau  -1      0
                              0      1     0    0   tau     1
                              0      C3    0    0   C2   tau - C1 )];

(* kappa can invertible *)
FullSimplify[emDet[kappa]]

(* kappa has axion component if and only if tau = 0*)
FullSimplify[emTrace[kappa]]

(* Since this does not simplify to {0}, kappa has a skewon component *)
Union[Flatten[kappa - emPoincare[kappa]]]

(* Since this does not simplify to {0}, kappa has a principal component *)
Union[Flatten[kappa + emPoincare[kappa]]]
```

$-\text{tau}\,(\text{C2} + (\text{C1} - \text{tau})\,\text{tau})\,\left(-1 + \text{tau}^3 + \text{C1}\,\text{tau}\,(1 + \text{tau})\right)$

6 tau

$\left\{-1, 0, 1, -\text{C1}, \text{C1}, 1 + \text{C1}, 1 - \text{C2}, \text{C3}, \frac{1}{4}\,(-4\,\text{C1} - 4\,\text{tau}) + \text{tau}, \frac{1}{4}\,(4\,\text{C1} - 4\,\text{tau}) + \text{tau}\right\}$

$\left\{-2, -1, 0, 1, 1 - \text{C1}, -1 + \text{C1}, -1 - \text{C2}, 1 + \text{C2}, -\text{C3}, \text{C3}, 2\,\text{tau}, -\text{C1} + 2\,\text{tau}, \text{C1} + 2\,\text{tau}, \text{tau} + \frac{1}{4}\,(-4\,\text{C1} + 4\,\text{tau}), \text{tau} + \frac{1}{4}\,(4\,\text{C1} + 4\,\text{tau})\right\}$

- **Define parameters to show that kappa is algebraically decomposable**

$$\text{Abivector} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix};$$

$$\text{Bbivector} = \begin{pmatrix} 0 & 0 & C1 + C3 & C1 \\ 0 & 0 & 0 & C1 - C2 \\ -(C1 + C3) & 0 & 0 & -1 \\ -C1 & -(C1 - C2) & 1 & 0 \end{pmatrix};$$

```
alpha = tau^2 - 1;
beta = -tau;
gamma = 1;
rho = 1 / 2;

(*  Verify that kappa satisfies equation (44)   *)
RHS = alpha emIdentityKappa[] +
   beta (kappa + emPoincare[kappa]) + gamma emCompose[emPoincare[kappa], kappa];
LHS = emBiProduct[rho, Abivector, Bbivector] + emBiProduct[rho, Bbivector, Abivector];
eqs = Union[Flatten[RHS - LHS]];
Union[Simplify[eqs]]
```

```
{0}
```

- **Write equations that should be satisfied for D**

```
(* contract kappa with a bivector from the left *)
contract[biv_, kappa_] := Table[
  1 / 2 Sum[
    biv[[i]][[j]] emReadNormal[kappa, a, b, i, j]
    ,
    {i, 1, 4}, {j, 1, 4}
   ]
  ,
  {a, 1, 4}, {b, 1, 4}
 ]
```

- **There is no bivector D that satisfies**

D (kappa + beta Id) = 1/2 trace (rho bar(D) otimes D) A + B.

```
Dbivector = emMatrix["d", 4, Structure → "AntiSymmetric"];
dLHS = contract[Dbivector, kappa + beta emIdentityKappa[]];
dRHS = 1 / 2 emTrace[emBiProduct[rho, Dbivector, Dbivector]] Abivector + Bbivector;
deqs = Union[Flatten[dLHS - dRHS]];

deqs
```

$$\Big\{-1,\ 0,\ 1,\ -C1 - C3 + \frac{1}{2}\,(-2\,d12 + 2\,C3\,d23 - 2\,d24),\ \frac{1}{2}\,(-2\,C1\,d23 - 2\,d24),$$
$$\frac{1}{2}\,(2\,C1\,d23 + 2\,d24),\ C1 + C3 + \frac{1}{2}\,(2\,d12 - 2\,C3\,d23 + 2\,d24),\ C1 + \frac{1}{2}\,(-2\,d13 - 2\,d34),$$
$$C1 - C2 + \frac{1}{2}\,(2\,C2\,d23 - 2\,d34),\ -C1 + \frac{1}{2}\,(2\,d13 + 2\,d34),\ -C1 + C2 + \frac{1}{2}\,(-2\,C2\,d23 + 2\,d34),$$
$$\frac{1}{2}\,(2\,C1\,d12 + 2\,C1\,d13 + 2\,d14) + \frac{1}{2}\,(-2\,d14\,d23 + 2\,d13\,d24 - 2\,d12\,d34),$$
$$\frac{1}{2}\,(-2\,C1\,d12 - 2\,C1\,d13 - 2\,d14) + \frac{1}{2}\,(2\,d14\,d23 - 2\,d13\,d24 + 2\,d12\,d34)\Big\}$$

- **Fresnel surface factorises**

```
coords = {xi0, xi1, xi2, xi3};
fr = FullSimplify[emKappaToFresnel[kappa, coords]];
```

```
Mat = ( 0              C1 + C3           -2 C1 - C3   -C2 - C1 C3
        C1 + C3        0                 -C1 + C2     1 + C1^2 - C1 C2
        -2 C1 - C3     -C1 + C2          2            -C1
        -C2 - C1 C3    1 + C1^2 - C1 C2  -C1          -2 C2 );
LinFactors = xi0 (xi0 + xi1 + xi3);
Simplify[fr - 1 / 2 LinFactors (coords.Mat.coords)]
```

0

- **For suitable choices of C1,C2,C3, the quadratic factor has a Lorentz signature**

```
sub = {C1 → 0, C2 → 1 / 2, C3 → 1 / 2, tau → 2};

FullSimplify[Det[Mat /. sub]]
FullSimplify[emDet[kappa /. sub]]
```

$-\frac{3}{16}$

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