In[1]:=
```
SetDirectory["~/KappaLib"];
<< kappaLib-1.2.m
```

```
Loading KappaLib v1.2
```

- **Metaclass IV:**

In[3]:=
```
kappa = emMatrixToKappa[
  a1  0   0   -b1  0    0
  0   a2  0   0    -b2  0
  0   0   a3  0    0    a4
  b1  0   0   a1   0    0
  0   b2  0   0    a2   0
  0   0   a4  0    0    a3
];
```

- **By Theorem 3.5 we may assume that a2 = a1 and b2 = b1**

In[4]:=
```
sub = {a2 → a1, b2 → b1};
kappa = kappa //. sub;
```

- **Let us also assume that a1 != a3**
- **Medium characteristics**

In[6]:=
```
emKappaToMatrix[kappa] // MatrixForm
FullSimplify[emDet[kappa]]
Simplify[emTrace[kappa]]
```

Out[6]//MatrixForm=

$$\begin{pmatrix} a1 & 0 & 0 & -b1 & 0 & 0 \\ 0 & a1 & 0 & 0 & -b1 & 0 \\ 0 & 0 & a3 & 0 & 0 & a4 \\ b1 & 0 & 0 & a1 & 0 & 0 \\ 0 & b1 & 0 & 0 & a1 & 0 \\ 0 & 0 & a4 & 0 & 0 & a3 \end{pmatrix}$$

Out[7]= $(a3 - a4)\ (a3 + a4)\ \left(a1^2 + b1^2\right)^2$

Out[8]= $2\ (2\ a1 + a3)$

- **Define coefficients in Theorem 5.1:**

In[9]:=
```
rho = 1/(8 (a3 - a1) a4);
mu = -a1;
lambda = b1^2;
sigma = (a4^2 - (a3 - a1)^2)^2 + b1^2 (2 a4^2 + b1^2 + 2 (a3 - a1)^2);
```

$$\text{Abivector} = \begin{pmatrix} 0 & 0 & 0 & \left((a3 - a1)\,\hat{}\,2 \cdot\right. \\ 0 & 0 & 2\ (a3 - a1)\ a4 & \\ 0 & -2\ (a3 - a1)\ a4 & 0 & \\ -\left((a1 - a3)\,\hat{}\,2 + a4^2 + b1^2 + \sqrt{\text{sigma}}\right) & 0 & 0 & \end{pmatrix}$$

$$\text{Bbivector} = \begin{pmatrix} 0 & 0 & 0 & \left((a1 - a3)\,\hat{}\,2 + a\cdot\right. \\ 0 & 0 & 2\ (-a1 + a3)\ a4 & \\ 0 & 2\ (a1 - a3)\ a4 & 0 & \\ -\left((a1 - a3)\,\hat{}\,2 + a4^2 + b1^2 - \sqrt{\text{sigma}}\right) & 0 & 0 & \end{pmatrix}$$

```
Simplify[Abivector + Transpose[Abivector]]
Simplify[Bbivector + Transpose[Bbivector]]
```

Out[15]= {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}

Out[16]= {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}

- **Verify claim:**

In[17]:=
```
eta = kappa + mu emIdentityKappa[];
LHS = emCompose[eta, eta];
RHS = -lambda emIdentityKappa[] +
   emBiProduct[rho, Abivector, Bbivector] + emBiProduct[rho, Bbivector, Abivector];
Union[Flatten[Simplify[LHS - RHS]]]
```

Out[20]= {0}

- **Note:**
  *** kappa + mu Id is not trace-free in this case**
  *** lambda > 0**
  *** rho, A, B are all non-zero**

In[21]:=
```
emTrace[eta]
```

Out[21]= $-2\, a1 + 2\, a3$

Solvability of equation for D

- **Define constants alpha, beta, gamma that appear in definition of algebraically decomposable medium**

In[22]:=
```
alpha = lambda + mu^2;
beta = mu;
gamma = 1;
```

- **Explicitly verify that kappa is algebraically decomposable:**

In[25]:=
```
LHS = alpha emIdentityKappa[] +
   beta (kappa + emPoincare[kappa]) + gamma emCompose[emPoincare[kappa], kappa];
RHS = emBiProduct[rho, Abivector, Bbivector] + emBiProduct[rho, Bbivector, Abivector];
eqs = Union[Simplify[Flatten[(LHS - RHS)]]]
```

Out[27]= {0}

- **Existence of D:**

In[28]:=
```
Dbivector = 1/(-a1 + a3) Bbivector;
(* contract kappa with a bivector from the left *)
contract[biv_, kappa_] := Table[
  1 / 2 Sum[
    biv[[i]][[j]] emReadNormal[kappa, a, b, i, j]
    ,
    {i, 1, 4}, {j, 1, 4}
   ]
  ,
  {a, 1, 4}, {b, 1, 4}
 ]
dLHS = contract[Dbivector, kappa + beta emIdentityKappa[]];
dRHS = 1 / 2 emTrace[emBiProduct[rho, Dbivector, Dbivector]] Abivector + Bbivector;
deqs = Union[Flatten[Simplify[dLHS - dRHS]]]
```

Out[32]= {0}