In[1]:=
```
SetDirectory["~/KappaLib"];
<< kappaLib-1.2.m
<< helper.m
```

```
Loading KappaLib v1.2

Loading helper.m..
```

- **Metaclass I:**

In[4]:=

$$\texttt{kappa = emMatrixToKappa}\left[\begin{pmatrix} a1 & 0 & 0 & -b1 & 0 & 0 \\ 0 & a2 & 0 & 0 & -b2 & 0 \\ 0 & 0 & a3 & 0 & 0 & -b3 \\ b1 & 0 & 0 & a1 & 0 & 0 \\ 0 & b2 & 0 & 0 & a2 & 0 \\ 0 & 0 & b3 & 0 & 0 & a3 \end{pmatrix}\right];$$

- **By Theorem 3.5 we may assume that a2 = a3 and b2 = b3**

In[5]:=
```
sub = {a3 → a2, b3 → b2}
```

Out[5]=
```
{a3 → a2, b3 → b2}
```

- **In addition, let us assume that a1 != a2**

In[6]:=
```
kappa = kappa //. sub;
```

- **Medium characteristics**

In[7]:=
```
emKappaToMatrix[kappa] // MatrixForm
FullSimplify[emDet[kappa]]
Simplify[emTrace[kappa]]
```

Out[7]//MatrixForm=

$$\begin{pmatrix} a1 & 0 & 0 & -b1 & 0 & 0 \\ 0 & a2 & 0 & 0 & -b2 & 0 \\ 0 & 0 & a2 & 0 & 0 & -b2 \\ b1 & 0 & 0 & a1 & 0 & 0 \\ 0 & b2 & 0 & 0 & a2 & 0 \\ 0 & 0 & b2 & 0 & 0 & a2 \end{pmatrix}$$

Out[8]= $\left(a1^2 + b1^2\right) \left(a2^2 + b2^2\right)^2$

Out[9]= $2\,(a1 + 2\,a2)$

- **Define coefficients in Theorem 5.1:**

In[10]:=

$$\texttt{rho} = \frac{1}{8\,(a1 - a2)\,b1};$$

```
mu = - a2;
lambda = b2^2;
```

$$\texttt{Sigma} = \left((a1 - a2)^2 + (b1 - b2)^2\right)\left((a1 - a2)^2 + (b1 + b2)^2\right);$$

$$\texttt{Abivector} = \begin{pmatrix} 0 & 2\,(a1 - a2)\,b1 & 0 & 0 \\ -2\,(a1 - a2)\,b1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \left((a1 - a2)^2 - b1^2 \cdot \right. \\ 0 & 0 & -\left((a1 - a2)^2 - b1^2 + b2^2 + \sqrt{\texttt{Sigma}}\right) & 0 \end{pmatrix}$$

$$\texttt{Bbivector} = \begin{pmatrix} 0 & 2\,(a1 - a2)\,b1 & 0 \\ 2\,(-a1 + a2)\,b1 & 0 & 0 \\ 0 & 0 & 0 & \left((a1 - a2)^2 - b1^2\right. \\ 0 & 0 & -\left((a1 - a2)^2 - b1^2 + b2^2 - \sqrt{\texttt{Sigma}}\right) \end{pmatrix}$$

In[16]:=
```
Abivector + Transpose[Abivector]
Simplify[Bbivector + Transpose[Bbivector]]
```

Out[16]= {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}

Out[17]= {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}

- **Verify claim:**

In[18]:=
```
eta = kappa + mu emIdentityKappa[];
LHS = emCompose[eta, eta];
RHS = -lambda emIdentityKappa[] +
   emBiProduct[rho, Abivector, Bbivector] + emBiProduct[rho, Bbivector, Abivector];
Union[Flatten[Simplify[LHS - RHS]]]
```

Out[21]= {0}

- **Note:**
  *** kappa + mu Id is not trace-free**
  *** lambda > 0**
  *** rho, A, B are all non-zero**

In[22]:=
```
emTrace[eta]
```

Out[22]= 2 a1 - 2 a2

Solvability of equation for D

- **Define constants alpha, beta, gamma that appear in definition of algebraically decomposable medium**

In[23]:=
```
alpha = lambda + mu^2;
beta = mu;
gamma = 1;
```

- **Explicitly verify that kappa is algebraically decomposable:**

In[26]:=
```
LHS = alpha emIdentityKappa[] +
   beta (kappa + emPoincare[kappa]) + gamma emCompose[emPoincare[kappa], kappa];
RHS = emBiProduct[rho, Abivector, Bbivector] + emBiProduct[rho, Bbivector, Abivector];
eqs = Union[Simplify[Flatten[(LHS - RHS)]]]
```

Out[28]= {0}

- **Existence of D:**

In[29]:=
```
Dbivector = 1/(a1 - a2) Bbivector;

(* contract kappa with a bivector from the left *)
contract[biv_, kappa_] := Table[
  1 / 2 Sum[
    biv[[i]][[j]] emReadNormal[kappa, a, b, i, j]
    ,
    {i, 1, 4}, {j, 1, 4}
   ]
  ,
  {a, 1, 4}, {b, 1, 4}
 ]
dLHS = contract[Dbivector, kappa + beta emIdentityKappa[]];
dRHS = 1 / 2 emTrace[emBiProduct[rho, Dbivector, Dbivector]] Abivector + Bbivector;
deqs = Union[Flatten[Simplify[dLHS - dRHS]]]
```

Out[33]= {0}