

```
In[1]:= SetDirectory["/www/user/fdahl/papers/Conjugation/"];
<< kappaLib.m
<< Petrov.m
```

KappaLib v1.1

Petrov routine loaded

$$\text{In[4]:= } \mathbf{B} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix};$$

$$\text{In[5]:= } \mathbf{V} = \begin{pmatrix} \text{lam1} & 1 & 0 & 0 & 0 & 0 \\ 0 & \text{lam1} & 1 & 0 & 0 & 0 \\ 0 & 0 & \text{lam1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \text{lam2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \text{sigma1} & \text{tau1} \\ 0 & 0 & 0 & 0 & -\text{tau1} & \text{sigma1} \end{pmatrix};$$

$$\text{In[6]:= } \mathbf{W} = \begin{pmatrix} 0 & 0 & \text{eps1} & 0 & 0 & 0 \\ 0 & \text{eps1} & 0 & 0 & 0 & 0 \\ \text{eps1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \text{eps2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} /. \{\text{eps2} \rightarrow -\text{eps1}\};$$

■ Solve S-matrix manually for metaclass XXII

$$\text{In[7]:= petrovTarget} = \begin{pmatrix} -\text{tau1} & 0 & 0 & 0 & 0 & \text{sigma1} \\ 0 & 0 & \frac{\text{eps1}}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \text{lam1} & 0 \\ 0 & \frac{\text{eps1}}{\sqrt{2}} & \text{eps1} \frac{\text{lam1}-\text{lam2}}{2} & \frac{\text{lam1}+\text{lam2}}{2} & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{\text{lam1}+\text{lam2}}{2} & \text{eps1} \frac{\text{lam1}-\text{lam2}}{2} & 0 & 0 \\ 0 & \text{lam1} & 0 & 0 & 0 & 0 \\ \text{sigma1} & 0 & 0 & 0 & 0 & \text{tau1} \end{pmatrix};$$

```
mat = Table[ToExpression["x" <> ToString[i] <> ToString[j]], {i, 1, 6}, {j, 1, 6}];
Petrov[mat]
```

$$\text{j3} = \text{Flatten} \left[ \begin{pmatrix} \text{x14} & \text{x15} & \text{x16} & \text{x13} & \text{x12} & \text{x11} \\ \text{x24} & \text{x25} & \text{x26} & \text{x23} & \text{x22} & \text{x21} \\ \text{x34} & \text{x35} & \text{x36} & \text{x33} & \text{x32} & \text{x31} \\ \text{x64} & \text{x65} & \text{x66} & \text{x63} & \text{x62} & \text{x61} \\ \text{x54} & \text{x55} & \text{x56} & \text{x53} & \text{x52} & \text{x51} \\ \text{x44} & \text{x45} & \text{x46} & \text{x43} & \text{x42} & \text{x41} \end{pmatrix} - \text{petrovTarget} \right];$$

```
eqs = Table[j3[[i]] == 0, {i, 1, Length[j3]}];
sol = Solve[eqs, Variables[mat]];
mat = mat //. sol[[1]];
```

Out[8]/MatrixForm=

$$\begin{pmatrix} \text{x14} & \text{x15} & \text{x16} & \text{x13} & \text{x12} & \text{x11} \\ \text{x24} & \text{x25} & \text{x26} & \text{x23} & \text{x22} & \text{x21} \\ \text{x34} & \text{x35} & \text{x36} & \text{x33} & \text{x32} & \text{x31} \\ \text{x64} & \text{x65} & \text{x66} & \text{x63} & \text{x62} & \text{x61} \\ \text{x54} & \text{x55} & \text{x56} & \text{x53} & \text{x52} & \text{x51} \\ \text{x44} & \text{x45} & \text{x46} & \text{x43} & \text{x42} & \text{x41} \end{pmatrix}$$

```
In[13]:= mat // MatrixForm
```

Out[13]/MatrixForm=

$$\begin{pmatrix} \text{sigma1} & 0 & 0 & -\text{tau1} & 0 & 0 \\ 0 & \text{lam1} & \frac{1}{\sqrt{2}} & 0 & 0 & \frac{\text{eps1}}{\sqrt{2}} \\ 0 & 0 & \frac{\text{lam1}+\text{lam2}}{2} & 0 & \frac{\text{eps1}}{\sqrt{2}} & \frac{1}{2} \text{eps1} (\text{lam1} - \text{lam2}) \\ \text{tau1} & 0 & 0 & \text{sigma1} & 0 & 0 \\ 0 & 0 & 0 & 0 & \text{lam1} & 0 \\ 0 & 0 & \frac{1}{2} \text{eps1} (\text{lam1} - \text{lam2}) & 0 & \frac{1}{\sqrt{2}} & \frac{\text{lam1}+\text{lam2}}{2} \end{pmatrix}$$

```
In[14]:= S = Table[ToExpression["s" <> ToString[i] <> ToString[j]], {i, 1, 6}, {j, 1, 6}];
j1 = Flatten[Transpose[S].B.S - W];
j2 = Flatten[mat.S - S.V];
```

```
In[17]:= exp = Join[j1, j2];
```

```
In[18]:= sub = {s51 → 0, s52 → 0, s54 → 0, s16 → s45, s15 → -s46, s31 → -eps1 s61, s61 → 0,
  s62 → eps1 s32, s64 → -eps1 s34, s56 → 0, s55 → 0, s41 → 0, s11 → 0, s35 → 0,
  s36 → 0, s46 → 0, s65 → 0, s12 → 0, s42 → 0, s66 → 0, s14 → 0, s44 → 0, s43 → 0,
  s13 → 0, s25 → 0, s26 → 0, s24 → 0, s45 → 1, s21 → 1, s53 → eps1, s32 → 1 / Sqrt[2],
  s63 → eps1 s33, s22 → -Sqrt[2] s33, s33 → 0, s23 → 0, s34 → 1 / Sqrt[2]};
Simplify[Sort[Union[exp //. sub], emSize[#1] ≤ emSize[#2] &]] // MatrixForm
Simplify[s //. sub] // MatrixForm
```

Out[19]//MatrixForm=

$$\begin{pmatrix} 0 \\ \frac{1}{2} (1 - \text{eps1}^2) \\ \frac{1}{2} (-1 + \text{eps1}^2) \\ \frac{-1 + \text{eps1}^2}{\sqrt{2}} \\ -\frac{(-1 + \text{eps1}^2) (\text{lam1} - \text{lam2})}{2 \sqrt{2}} \\ \frac{(-1 + \text{eps1}^2) (\text{lam1} - \text{lam2})}{2 \sqrt{2}} \\ 0 \\ 0 \end{pmatrix}$$

Out[20]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \text{eps1} & 0 & 0 & 0 \\ 0 & \frac{\text{eps1}}{\sqrt{2}} & 0 & -\frac{\text{eps1}}{\sqrt{2}} & 0 & 0 \end{pmatrix}$$

```
In[21]:= (* export .pdf *)
NotebookPrint[SelectedNotebook[],
  "/www/user/fdahl/papers/Conjugation/notebooks/ClassXXII_Solve.pdf"]
```