

```
In[1]:= SetDirectory["/www/user/fdahl/papers/Conjugation/"];
<< kappaLib.m
<< Petrov.m
```

KappaLib v1.1

Petrov routine loaded

■ **Class XXIII: (31 11)**

$$\text{In[4]:= } \mathbf{B} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix};$$

$$\text{In[5]:= } \mathbf{V} = \begin{pmatrix} \text{lam1} & 1 & 0 & 0 & 0 & 0 \\ 0 & \text{lam1} & 1 & 0 & 0 & 0 \\ 0 & 0 & \text{lam1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \text{lam2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \text{lam3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \text{lam4} \end{pmatrix};$$

```
In[6]:= Eigenvalues[V]
```

```
Out[6]= {lam1, lam1, lam1, lam2, lam3, lam4}
```

$$\text{In[7]:= } \mathbf{W} = \begin{pmatrix} 0 & 0 & \text{eps1} & 0 & 0 & 0 \\ 0 & \text{eps1} & 0 & 0 & 0 & 0 \\ \text{eps1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \text{eps2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \text{eps3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \text{eps4} \end{pmatrix};$$

■ **eps2,eps3,eps4 have same block size, so we may assume that eps2 <= eps3 <= eps4.**

```
In[8]:= Eigenvalues[W]
```

```
Out[8]= {-eps1, eps1, eps1, eps2, eps3, eps4}
```

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In[9]:= Sort[Eigenvalues[W] /. {eps1 → -1, eps2 → -1, eps3 → -1, eps4 → -1}]
Sort[Eigenvalues[W] /. {eps1 → -1, eps2 → -1, eps3 → -1, eps4 → 1}]
Sort[Eigenvalues[W] /. {eps1 → -1, eps2 → -1, eps3 → 1, eps4 → 1}]
Sort[Eigenvalues[W] /. {eps1 → -1, eps2 → 1, eps3 → 1, eps4 → 1}]

Sort[Eigenvalues[W] /. {eps1 → 1, eps2 → -1, eps3 → -1, eps4 → -1}]
Sort[Eigenvalues[W] /. {eps1 → 1, eps2 → -1, eps3 → -1, eps4 → 1}]
Sort[Eigenvalues[W] /. {eps1 → 1, eps2 → -1, eps3 → 1, eps4 → 1}]
Sort[Eigenvalues[W] /. {eps1 → 1, eps2 → 1, eps3 → 1, eps4 → 1}]

Out[9]= {-1, -1, -1, -1, -1, 1}
Out[10]= {-1, -1, -1, -1, 1, 1}
Out[11]= {-1, -1, -1, 1, 1, 1}
Out[12]= {-1, -1, 1, 1, 1, 1}
Out[13]= {-1, -1, -1, -1, 1, 1}
Out[14]= {-1, -1, -1, 1, 1, 1}
Out[15]= {-1, -1, 1, 1, 1, 1}
Out[16]= {-1, 1, 1, 1, 1, 1}

```

■ 2 sign possibilities: --++, +--+

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In[17]:= pos1 = {eps1 → -1, eps2 → -1, eps3 → 1, eps4 → 1};
pos2 = {eps1 → 1, eps2 → -1, eps3 → -1, eps4 → 1};

In[19]:= Eigenvalues[W /. pos1]
Eigenvalues[W /. pos2]

Out[19]= {-1, -1, -1, 1, 1, 1}
Out[20]= {-1, -1, -1, 1, 1, 1}

```

■ For both configurations, eps2=-1, eps4=1 and eps3 = -eps1

```

In[21]:= W = W /. {eps2 → -1, eps4 → 1, eps3 → -eps1};

In[22]:= W // MatrixForm

```

```

Out[22]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & \text{eps1} & 0 & 0 & 0 \\ 0 & \text{eps1} & 0 & 0 & 0 & 0 \\ \text{eps1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\text{eps1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$


```

$$\text{In[23]:= } \mathbf{S} = \begin{pmatrix} 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{\text{eps1}}{\sqrt{2}} & 0 & 0 & \frac{-\text{eps1}}{\sqrt{2}} & 0 \\ \text{eps1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix};$$

■ Check that S is in the set `mathcal{S}`

`In[24]:= Transpose[S].B.S == W`

`Out[24]= True`

■ Compute result

`In[25]:= res = S.V.Inverse[S];
r = Simplify[res];
r // MatrixForm`

`Out[27]//MatrixForm=`

$$\begin{pmatrix} \frac{\text{lam2}+\text{lam4}}{2} & 0 & 0 & \frac{1}{2}(-\text{lam2}+\text{lam4}) & 0 & 0 \\ 0 & \frac{\text{lam1}+\text{lam3}}{2} & 0 & 0 & \frac{1}{2}\text{eps1}(\text{lam1}-\text{lam3}) & \frac{\text{eps1}}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \text{lam1} & 0 & \frac{\text{eps1}}{\sqrt{2}} & 0 \\ \frac{1}{2}(-\text{lam2}+\text{lam4}) & 0 & 0 & \frac{\text{lam2}+\text{lam4}}{2} & 0 & 0 \\ 0 & \frac{\text{lam1}-\text{lam3}}{2\text{eps1}} & 0 & 0 & \frac{\text{lam1}+\text{lam3}}{2} & \frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & 0 & \text{lam1} \end{pmatrix}$$

`In[28]:= Petrov[r]`

`Out[28]//MatrixForm=`

$$\begin{pmatrix} \frac{1}{2}(-\text{lam2}+\text{lam4}) & 0 & 0 & 0 & 0 & \frac{\text{lam2}+\text{lam4}}{2} \\ 0 & \frac{1}{2}\text{eps1}(\text{lam1}-\text{lam3}) & \frac{\text{eps1}}{\sqrt{2}} & 0 & \frac{\text{lam1}+\text{lam3}}{2} & 0 \\ 0 & \frac{\text{eps1}}{\sqrt{2}} & 0 & \text{lam1} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & \text{lam1} & 0 & 0 & 0 \\ 0 & \frac{\text{lam1}+\text{lam3}}{2} & \frac{1}{\sqrt{2}} & 0 & \frac{\text{lam1}-\text{lam3}}{2\text{eps1}} & 0 \\ \frac{\text{lam2}+\text{lam4}}{2} & 0 & 0 & 0 & 0 & \frac{1}{2}(-\text{lam2}+\text{lam4}) \end{pmatrix}$$

■ Export notebook as .pdf

`In[29]:= NotebookPrint[SelectedNotebook[],
"/www/user/fdahl/papers/Conjugation/notebooks/ClassXXIII.pdf"]`