In[1]:=
```
SetDirectory["/www/user/fdahl/papers/Conjugation/"];
<< kappaLib.m
<< Petrov.m
```

```
KappaLib v1.1

Petrov routine loaded
```

- **Class XVIII: (2 1111)**

In[4]:=
$$B = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix};$$

In[5]:=
$$V = \begin{pmatrix} lam1 & 1 & 0 & 0 & 0 & 0 \\ 0 & lam1 & 0 & 0 & 0 & 0 \\ 0 & 0 & lam2 & 0 & 0 & 0 \\ 0 & 0 & 0 & lam3 & 0 & 0 \\ 0 & 0 & 0 & 0 & lam4 & 0 \\ 0 & 0 & 0 & 0 & 0 & lam5 \end{pmatrix};$$

In[6]:=
```
Eigenvalues[V]
```

Out[6]=
```
{lam1, lam1, lam2, lam3, lam4, lam5}
```

In[7]:=
$$W = \begin{pmatrix} 0 & eps1 & 0 & 0 & 0 & 0 \\ eps1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & eps2 & 0 & 0 & 0 \\ 0 & 0 & 0 & eps3 & 0 & 0 \\ 0 & 0 & 0 & 0 & eps4 & 0 \\ 0 & 0 & 0 & 0 & 0 & eps5 \end{pmatrix};$$

In[8]:=
```
Eigenvalues[W]
```

Out[8]=
```
{-eps1, eps1, eps2, eps3, eps4, eps5}
```

- **We may assume that eps2 <= esp3 <= esp4 <= eps5**

In[9]:=
```
Wp = W /. eps1 → 1;
Sort[Eigenvalues[Wp] /. {eps2 → -1, eps3 → -1, eps4 → -1, eps5 → -1}]
Sort[Eigenvalues[Wp] /. {eps2 → -1, eps3 → -1, eps4 → -1, eps5 → 1}]
Sort[Eigenvalues[Wp] /. {eps2 → -1, eps3 → -1, eps4 → 1, eps5 → 1}]
Sort[Eigenvalues[Wp] /. {eps2 → -1, eps3 → 1, eps4 → 1, eps5 → 1}]
Sort[Eigenvalues[Wp] /. {eps2 → 1, eps3 → 1, eps4 → 1, eps5 → 1}]
```

Out[10]=
```
{-1, -1, -1, -1, -1, 1}
```

Out[11]=
```
{-1, -1, -1, -1, 1, 1}
```

Out[12]=
```
{-1, -1, -1, 1, 1, 1}
```

Out[13]=
```
{-1, -1, 1, 1, 1, 1}
```

Out[14]=
```
{-1, 1, 1, 1, 1, 1}
```

In[15]:=
```
Wm = W /. eps1 → -1;
Sort[Eigenvalues[Wm] /. {eps2 → -1, eps3 → -1, eps4 → -1, eps5 → -1}]
Sort[Eigenvalues[Wm] /. {eps2 → -1, eps3 → -1, eps4 → -1, eps5 → 1}]
Sort[Eigenvalues[Wm] /. {eps2 → -1, eps3 → -1, eps4 → 1, eps5 → 1}]
Sort[Eigenvalues[Wm] /. {eps2 → -1, eps3 → 1, eps4 → 1, eps5 → 1}]
Sort[Eigenvalues[Wm] /. {eps2 → 1, eps3 → 1, eps4 → 1, eps5 → 1}]
```

Out[16]= $\{-1, -1, -1, -1, -1, 1\}$

Out[17]= $\{-1, -1, -1, -1, 1, 1\}$

Out[18]= $\{-1, -1, -1, 1, 1, 1\}$

Out[19]= $\{-1, -1, 1, 1, 1, 1\}$

Out[20]= $\{-1, 1, 1, 1, 1, 1\}$

■ We see: eps2, eps3, eps4, eps5 are fixed, but eps1 is arbitrary:

In[21]:=
```
W = W /. {eps2 → -1, eps3 → -1, eps4 → 1, eps5 → 1};
Eigenvalues[W]
```

Out[22]= $\{-1, -1, 1, 1, -\text{eps1}, \text{eps1}\}$

In[23]:=
```
(* Found using FindSPermutations *)
```

$$S = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \text{eps1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix};$$

■ Check that S is in the set mathcal(S)

In[24]:=
```
Transpose[S].B.S == W
```

Out[24]= True

■ Compute result

In[25]:=
```
res = S.V.Inverse[S];
res // MatrixForm
```

Out[26]//MatrixForm=

$$\begin{pmatrix} \text{lam1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\text{lam2}}{2} + \frac{\text{lam4}}{2} & 0 & 0 & -\frac{\text{lam2}}{2} + \frac{\text{lam4}}{2} & 0 \\ 0 & 0 & \frac{\text{lam3}}{2} + \frac{\text{lam5}}{2} & 0 & 0 & -\frac{\text{lam3}}{2} + \frac{\text{lam5}}{2} \\ \text{eps1} & 0 & 0 & \text{lam1} & 0 & 0 \\ 0 & -\frac{\text{lam2}}{2} + \frac{\text{lam4}}{2} & 0 & 0 & \frac{\text{lam2}}{2} + \frac{\text{lam4}}{2} & 0 \\ 0 & 0 & -\frac{\text{lam3}}{2} + \frac{\text{lam5}}{2} & 0 & 0 & \frac{\text{lam3}}{2} + \frac{\text{lam5}}{2} \end{pmatrix}$$

In[27]:=
```
Petrov[res]
```

Out[27]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \text{lam1} \\ 0 & \frac{1}{2}\,(-\text{lam2}+\text{lam4}) & 0 & 0 & \frac{\text{lam2}+\text{lam4}}{2} & 0 \\ 0 & 0 & \frac{1}{2}\,(-\text{lam3}+\text{lam5}) & \frac{\text{lam3}+\text{lam5}}{2} & 0 & 0 \\ 0 & 0 & \frac{\text{lam3}+\text{lam5}}{2} & \frac{1}{2}\,(-\text{lam3}+\text{lam5}) & 0 & 0 \\ 0 & \frac{\text{lam2}+\text{lam4}}{2} & 0 & 0 & \frac{1}{2}\,(-\text{lam2}+\text{lam4}) & 0 \\ \text{lam1} & 0 & 0 & 0 & 0 & \text{eps1} \end{pmatrix}$$

■ Export notebook as .pdf

In[28]:=
```
NotebookPrint[SelectedNotebook[],
 "/www/user/fdahl/papers/Conjugation/notebooks/ClassXVIII.pdf"]
```