

■ Section 3.1: In the paper we use that

$$G^{ijkl} x_i x_j x_k x_l = G^{ijkl} x_i x_j x_l x_k$$

for the Tamm-Rubilar tensor density G^{ijkl} and the corresponding non-symmetrised version.

In this notebook we check this result.

```
In[1]:= p = Permutations[{x1, x2, x3, x4}]
```

```
Out[1]:= {{x1, x2, x3, x4}, {x1, x2, x4, x3}, {x1, x3, x2, x4}, {x1, x3, x4, x2}, {x1, x4, x2, x3},
{x1, x4, x3, x2}, {x2, x1, x3, x4}, {x2, x1, x4, x3}, {x2, x3, x1, x4}, {x2, x3, x4, x1},
{x2, x4, x1, x3}, {x2, x4, x3, x1}, {x3, x1, x2, x4}, {x3, x1, x4, x2}, {x3, x2, x1, x4},
{x3, x2, x4, x1}, {x3, x4, x1, x2}, {x3, x4, x2, x1}, {x4, x1, x2, x3}, {x4, x1, x3, x2},
{x4, x2, x1, x3}, {x4, x2, x3, x1}, {x4, x3, x1, x2}, {x4, x3, x2, x1}}
```

```
In[2]:= Length[p]
```

```
Out[2]:= 24
```

```
In[3]:= G[i_, j_, k_, l_] := Module[
  {tmp, perm, x1, x2, x3, x4},

  p = Permutations[{x1, x2, x3, x4}];
  p = p /. {x1 -> i, x2 -> j, x3 -> k, x4 -> l};

  1 / 4! Sum[f[p[[s]][[1]], p[[s]][[2]], p[[s]][[3]], p[[s]][[4]]], {s, 1, Length[p]}]
]
```

```
In[4]:= vars = {xi1, xi2, xi3, xi4};
```

```
symmetricSum = Sum[G[a1, a2, a3, a4] vars[[a1]] vars[[a2]] vars[[a3]] vars[[a4]],
  {a1, 1, 4}, {a2, 1, 4}, {a3, 1, 4}, {a4, 1, 4}];
```

```
In[6]:= nonSymmetricSum = Sum[f[a1, a2, a3, a4] vars[[a1]] vars[[a2]] vars[[a3]] vars[[a4]],
  {a1, 1, 4}, {a2, 1, 4}, {a3, 1, 4}, {a4, 1, 4}];
```

```
In[7]:= Simplify[symmetricSum - nonSymmetricSum]
```

```
Out[7]:= 0
```