▪ Proof of Proposition 3.5 (i): We compute spectrum of 4x4 matrix H

In[1]:=
```
(* Define metric (and inverse) *)
GG = DiagonalMatrix[{s1, s2, s3, s4}];
GG // MatrixForm
```

Out[2]//MatrixForm=

$$\begin{pmatrix} s1 & 0 & 0 & 0 \\ 0 & s2 & 0 & 0 \\ 0 & 0 & s3 & 0 \\ 0 & 0 & 0 & s4 \end{pmatrix}$$

In[3]:=
```
Ginv = Inverse[GG];
Ginv // MatrixForm
```

Out[4]//MatrixForm=

$$\begin{pmatrix} \frac{1}{s1} & 0 & 0 & 0 \\ 0 & \frac{1}{s2} & 0 & 0 \\ 0 & 0 & \frac{1}{s3} & 0 \\ 0 & 0 & 0 & \frac{1}{s4} \end{pmatrix}$$

In[5]:=
```
(* define covector *)
xi = {xi0, xi1, xi2, xi3};
```

In[6]:=
```
(* define H matrix *)
HH = (xi.Ginv.xi) Ginv - Table[ (xi.Ginv)[[i]] (xi.Ginv)[[j]], {i, 1, 4}, {j, 1, 4}];
HH // MatrixForm
```

Out[7]//MatrixForm=

$$\begin{pmatrix} -\frac{xi0^2}{s1^2}+\frac{\frac{xi0^2}{s1}+\frac{xi1^2}{s2}+\frac{xi2^2}{s3}+\frac{xi3^2}{s4}}{s1} & -\frac{xi0\,xi1}{s1\,s2} & -\frac{xi0\,xi2}{s1\,s3} & -\frac{xi0\,xi3}{s1\,s4} \\ -\frac{xi0\,xi1}{s1\,s2} & -\frac{xi1^2}{s2^2}+\frac{\frac{xi0^2}{s1}+\frac{xi1^2}{s2}+\frac{xi2^2}{s3}+\frac{xi3^2}{s4}}{s2} & -\frac{xi1\,xi2}{s2\,s3} & -\frac{xi1\,xi3}{s2\,s4} \\ -\frac{xi0\,xi2}{s1\,s3} & -\frac{xi1\,xi2}{s2\,s3} & -\frac{xi2^2}{s3^2}+\frac{\frac{xi0^2}{s1}+\frac{xi1^2}{s2}+\frac{xi2^2}{s3}+\frac{xi3^2}{s4}}{s3} & -\frac{xi2\,xi3}{s3\,s4} \\ -\frac{xi0\,xi3}{s1\,s4} & -\frac{xi1\,xi3}{s2\,s4} & -\frac{xi2\,xi3}{s3\,s4} & -\frac{xi3^2}{s4^2}+\frac{\frac{xi0^2}{s1}+\frac{xi1^2}{s2}+\frac{xi2^2}{s3}+\frac{xi3^2}{s4}}{s4} \end{pmatrix}$$

▪ Signature (+,+,+,+)

In[8]:=
```
Eigenvalues[HH /. {s1 → 1, s2 → 1, s3 → 1, s4 → 1}]
```

Out[8]= $\{0,\ xi0^2+xi1^2+xi2^2+xi3^2,\ xi0^2+xi1^2+xi2^2+xi3^2,\ xi0^2+xi1^2+xi2^2+xi3^2\}$

▪ Signature (-,+,+,+)

In[9]:=
```
Eigenvalues[HH /. {s1 → -1, s2 → 1, s3 → 1, s4 → 1}]
```

Out[9]= $\{0,\ -xi0^2-xi1^2-xi2^2-xi3^2,\ -xi0^2+xi1^2+xi2^2+xi3^2,\ -xi0^2+xi1^2+xi2^2+xi3^2\}$

▪ Signature (-,-,+,+)

In[10]:=
```
Eigenvalues[HH /. {s1 → -1, s2 → -1, s3 → 1, s4 → 1}]
```

Out[10]= $\{0,\ -xi0^2-xi1^2-xi2^2-xi3^2,\ xi0^2+xi1^2-xi2^2-xi3^2,\ -xi0^2-xi1^2+xi2^2+xi3^2\}$

▪ We do not need to consider the remaining signatures since matrix H is invariant under g -> -g.