In[1]:=
```
SetDirectory["~/writing/WIP/KappaLib/"];
<< KappaLib.m
```

```
KappaLib v1.1
```

## ■ Example 5.3: We show that there are complex medium that have the same Fresnel surface as the flat Minkowski metric

In[3]:=
```
Ax = {{-1/(2 z^2), 0, 0}, {0, -2 z, 0}, {0, 0, -z}};
Bx = -Ax;
Cx = {{I/(3 z^2) - I z, 0, 0}, {0, -I/(6 z^2) + I z, 0}, {0, 0, -I/(6 z^2)}};
Dx = Cx;
kappa = emABCDToKappa[Ax, Bx, Cx, Dx];
```

In[8]:=
```
Simplify[emKappaToFresnel[kappa, {xi0, xi1, xi2, xi3}]]
```

Out[8]=
$$-\left(-\text{xi0}^2+\text{xi1}^2+\text{xi2}^2+\text{xi3}^2\right)^2$$

In[9]:=
```
emTrace[kappa]
```

Out[9]=
$$0$$

In[10]:=
```
Simplify[emDet[kappa]]
```

Out[10]=
$$-\frac{\left(1+6\,z^3\right)^3\left(-5+126\,z^3-684\,z^6+648\,z^9\right)}{46\,656\,z^{12}}$$

## ■ Extra:

In[11]:=
```
Solve[(-5 + 126 z^3 - 684 z^6 + 648 z^9) == 0, z]
```

Out[11]=
$$\left\{\left\{z\to-\left(-\frac{5}{6}\right)^{1/3}\right\},\left\{z\to-\left(-\frac{1}{6}\right)^{1/3}\right\},\left\{z\to\left(\frac{5}{6}\right)^{1/3}\right\},\left\{z\to(-1)^{2/3}\left(\frac{5}{6}\right)^{1/3}\right\},\right.$$
$$\left.\left\{z\to-\frac{\left(-\frac{1}{2}\right)^{1/3}}{3^{2/3}}\right\},\left\{z\to\frac{1}{2^{1/3}\,3^{2/3}}\right\},\left\{z\to\frac{(-1)^{2/3}}{2^{1/3}\,3^{2/3}}\right\},\left\{z\to\frac{1}{6^{1/3}}\right\},\left\{z\to\frac{(-1)^{2/3}}{6^{1/3}}\right\}\right\}$$

In[12]:=
```
% // N
```

Out[12]=
```
{{z → -0.470518 - 0.814961 i}, {z → -0.275161 - 0.476592 i}, {z → 0.941036},
 {z → -0.470518 + 0.814961 i}, {z → -0.190786 - 0.330451 i}, {z → 0.381571},
 {z → -0.190786 + 0.330451 i}, {z → 0.550321}, {z → -0.275161 + 0.476592 i}}
```

## ■ One zero is z=6^(-1/3)

In[13]:=
```
kappa0 = kappa //. z → 1/6^(1/3);
```

In[14]:=
```
{A0, B0, C0, D0} = emKappaToABCD[kappa0];
```

In[15]:=
```
Simplify[A0] // MatrixForm
Simplify[C0] // MatrixForm
```

Out[15]//MatrixForm=

$$\begin{pmatrix} -\frac{3^{2/3}}{2^{1/3}} & 0 & 0 \\ 0 & -\frac{2^{2/3}}{3^{1/3}} & 0 \\ 0 & 0 & -\frac{1}{6^{1/3}} \end{pmatrix}$$

Out[16]//MatrixForm=

$$\begin{pmatrix} \frac{i}{6^{1/3}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{i}{6^{1/3}} \end{pmatrix}$$

In[17]:=
```
emDet[kappa0]
Simplify[emKappaToFresnel[kappa0, {xi0, xi1, xi2, xi3}]]
```

Out[17]= 0

Out[18]= $-\left(-\text{xi0}^2+\text{xi1}^2+\text{xi2}^2+\text{xi3}^2\right)^2$