■ Lemma 4.3: Computations for Claim 1, Case C

In[1]:=
```
SetDirectory["~/writing/WIP/KappaLib/"];
<< KappaLib.m
```

```
KappaLib v1.1
```

In[3]:=
```
Ax = DiagonalMatrix[{a1, 0, 0}];
Bx = emGeneralSymmetric3x3["B"];
Cx = emGeneral3x3["C"];
Dx = Transpose[Cx];
```

In[7]:=
```
eq1 = Flatten[Cx.Cx + Ax.Bx + IdentityMatrix[3]];
eq2 = Flatten[Bx.Cx + Transpose[Cx].Bx];
eq3 = Flatten[Cx.Ax + Ax.Transpose[Cx]];
eqs = Join[Join[eq1, eq2], eq3];
```

In[11]:=
```
Simplify[Union[eqs]] // MatrixForm
```

Out[11]//MatrixForm=

$$
\begin{pmatrix}
0 \\
2\,a1\,C11 \\
a1\,C21 \\
a1\,C31 \\
2\,(B11\,C11 + B12\,C21 + B13\,C31) \\
1 + a1\,B11 + C11^2 + C12\,C21 + C13\,C31 \\
C11\,C21 + C21\,C22 + C23\,C31 \\
B11\,C12 + B22\,C21 + B12\,(C11 + C22) + B23\,C31 + B13\,C32 \\
2\,(B12\,C12 + B22\,C22 + B23\,C32) \\
a1\,B12 + C11\,C12 + C12\,C22 + C13\,C32 \\
1 + C12\,C21 + C22^2 + C23\,C32 \\
B11\,C13 + B23\,C21 + B12\,C23 + B33\,C31 + B13\,(C11 + C33) \\
B13\,C12 + B12\,C13 + B23\,C22 + B22\,C23 + B33\,C32 + B23\,C33 \\
2\,(B13\,C13 + B23\,C23 + B33\,C33) \\
a1\,B13 + C11\,C13 + C12\,C23 + C13\,C33 \\
C13\,C21 + C23\,(C22 + C33) \\
C11\,C31 + C21\,C32 + C31\,C33 \\
C12\,C31 + C32\,(C22 + C33) \\
1 + C13\,C31 + C23\,C32 + C33^2
\end{pmatrix}
$$

■ Since a1 != 0, it follows that C11, C21, C31 = 0.

In[12]:=
```
subs = {};
subs = Append[subs, C11 → 0];
subs = Append[subs, C21 → 0];
subs = Append[subs, C31 → 0]; Simplify[Union[eqs //. subs]] // MatrixForm
```

Out[15]//MatrixForm=

$$
\begin{pmatrix}
0 \\
1 + a1\,B11 \\
B11\,C12 + B12\,C22 + B13\,C32 \\
2\,(B12\,C12 + B22\,C22 + B23\,C32) \\
a1\,B12 + C12\,C22 + C13\,C32 \\
1 + C22^2 + C23\,C32 \\
B11\,C13 + B12\,C23 + B13\,C33 \\
B13\,C12 + B12\,C13 + B23\,C22 + B22\,C23 + B33\,C32 + B23\,C33 \\
2\,(B13\,C13 + B23\,C23 + B33\,C33) \\
a1\,B13 + C12\,C23 + C13\,C33 \\
C23\,(C22 + C33) \\
C32\,(C22 + C33) \\
1 + C23\,C32 + C33^2
\end{pmatrix}
$$

- **Equation 1+ a1 B11 =0 implies that B11 != 0**
  Equation $1 + C23\, C32 + C33^2$ =0 implies that C23 != 0 and C32 != 0
- **Coordinate transformation**

In[16]:=
```
kappa = emABCDToKappa[Ax, Bx, Cx, Dx];
kappaSub = kappa //. subs;
emKappaToMatrix[kappaSub] // MatrixForm
```

Out[18]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & B11 & B12 & B13 \\ C12 & C22 & C32 & B12 & B22 & B23 \\ C13 & C23 & C33 & B13 & B23 & B33 \\ a1 & 0 & 0 & 0 & C12 & C13 \\ 0 & 0 & 0 & 0 & C22 & C23 \\ 0 & 0 & 0 & 0 & C32 & C33 \end{pmatrix}$$

- **Define Jacobian matrix for coordinate transformation**

  tilde x^0 = x^0 + x^3
  tilde x^i = x^i,   i=1,2,3.

  See emCoordinateChange in KappaLib source code.

In[19]:=
```
L = {
    {1, 0, 0, 1},
    {0, 1, 0, 0},
    {0, 0, 1, 0},
    {0, 0, 0, 1}
   };

kappaTrans = emCoordinateChange[kappaSub, L];
{Atrans, Btrans, Ctrans, Dtrans} = emKappaToABCD[kappaTrans];
Simplify[Atrans] // MatrixForm
Simplify[Det[Atrans]]
```

Out[22]//MatrixForm=

$$\begin{pmatrix} a1 + B22 + 2\,C12 & -B12 + C22 & C32 \\ -B12 + C22 & B11 & 0 \\ C32 & 0 & 0 \end{pmatrix}$$

Out[23]= $-B11\, C32^2$